Question Bank In Mathematics Class IX (Term II)

13 SURFACE AREAS AND VOLUMES

A. SUMMATIVE ASSESSMENT

13.1 SURFACE AREA OF A CUBOID AND A CUBE

1. Surface area of a cuboid of dimensions $l \times b \times h$ is $2(lb + bh + hl)$.

2. Lateral surface area of a cuboid of dimensions $l \times b \times h$ is $2h(l + b)$.

3. Surface area of a cube of edge a is $6a^2$.

4. Lateral surface area of a cube of edge a is $4a^2$.

5. Length of the diagonal of a cuboid of dimensions $l \times b \times h$ is $\sqrt{l^2 + b^2 + h^2}$.

6. Length of the diagonal of a cube of edge a is $a\sqrt{3}$.

TEXTBOOK'S EXERCISE 13.1

Q.1. *A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine :*

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring 1 m^2 costs Rs 20.

Sol. Here, $l = 1.5$ m, $b = 1.25$ m, $h = 65$ cm $= 0.65$ m.

Since the box is open at the top, it has only five faces.

(i) So, surface area of the box

$= lb + 2(bh + hl)$

$= 1.5 \times 1.25 \text{ m}^2 + 2\,(1.25 \times 0.65 + 0.65 \times 1.5) \text{ m}^2$

$= 1.875 \text{ m}^2 + 2\,(1.7875) \text{ m}^2$

$= (1.875 + 3.575) \text{ m}^2 = 5.45 \text{ m}^2$

Hence, 5.45 m^2 of sheet is required.

(ii) Cost of 1 m^2 of the sheet = Rs 20

$\therefore$ Cost of 5.45 m^2 of the sheet

= Rs $20 \times 5.45 \text{ m}^2$ = Rs 109

Q.2. *The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of whitewashing the walls of the room and the ceiling at the rate of Rs 7.50 per m^2.* **[2011 (T-II)]**

Sol. Here, $l = 5$ m, $b = 4$ m, $h = 3$ m

Surface area of the walls of the room and the ceiling $= 2h\,(l + b) + lb$

$= [2 \times 3\,(5 + 4) + 5 \times 4] \text{ m}^2$

$= (6 \times 9 + 20) \text{ m}^2 = 74 \text{ m}^2$

Cost of whitewashing = Rs 7.50 per m^2

$\therefore$ Total cost of whitewashing the walls and the ceiling of the room

= Rs 74×7.50 = Rs 555

Q.3. *The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs 10 per m^2 is Rs 15000, find the height of the hall.* **[2011 (T-II)]**

Sol. Let length, breadth and height of the hall be l, b and h respectively.

Perimeter of the floor of the hall

$= 2\,(l + b) = 250$ m.

Area of the four walls of the hall

$= 2h\,(l + b)$ ... (i)

Also, area of the four walls of the hall

$= \frac{15000}{10} \text{ m}^2 = 1500 \text{ m}^2$ ... (ii)

From (i) and (ii), we have

$2h\,(l + b) = 1500$

$\Rightarrow h \times 250 = 1500$ $[\because 2(l + b) = 250]$

$\Rightarrow \quad h = \frac{1500}{250} = 6$

Hence, height of the hall is 6 m

Q.4. *The paint in a certain container is sufficient to paint an area equal to* 9.375 m^2. *How many bricks of dimensions* 22.5 *cm* × 10 *cm* × 7.5 *cm can be painted out of this container?*

[2011 (T-II)]

Sol. Here, l = 22.5 cm, b = 10 cm, h = 7.5 cm.

Total surface area of 1 brick

$= 2\,(lb + bh + hl)$

$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$ cm^2

$= 2(225 + 75 + 168.75)$ cm^2 = 937.5 cm^2

$= \dfrac{937.5}{100 \times 100}$ m^2 = 0.09375 m^2.

∴ Required number of bricks

$= \dfrac{9.375}{0.09375} = 100$

Q.5. *A cubical box has each edge* 10 *cm and another cuboidal box is* 12.5 *cm long,* 10 *cm wide and* 8 *cm high.*

(i) *Which box has the greater lateral surface area and by how much?*

(ii) Which box has the smaller total surface area and by how much?

Sol. Here, a = 10 cm, l = 12.5 cm, b = 10 cm, h = 8 cm

(i) Lateral surface area of the cubical box

$= 4a^2 = 4 \times (10)^2$ cm^2 = 400 cm^2

Lateral surface area of the cuboidal box

$= 2h\,(l + b)$

$= 2 \times 8\,(12.5 + 10)$ cm^2

$= 16 \times 22.5$ cm^2 = 360 cm^2

Difference in the lateral surface areas of the two boxes = (400 – 360) cm^2 = 40 cm^2.

Hence, the cubical box has greater lateral surface area by 40 cm^2

(ii) Total surface area of the cubical box

$= 6a^2 = 6 \times (10)^2$ cm^2 = 600 cm^2

Total surface area of the cuboidal box

$= 2(lb + bh + hl)$

$= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)$ cm^2

$= 2(125 + 80 + 100)$ cm^2

$= 2 \times 305$ cm^2 = 610 cm^2

Difference in the total surface areas of the two boxes = (610 – 600) cm^2 = 10 cm^2

Hence, the cubical box has smaller total surface area by 10 cm^2

Q.6. *A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is* 30 *cm long,* 25 *cm wide and* 25 *cm high.*

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 *edges?*

Sol. Here, l = 30 cm, b = 25 cm, h = 25 cm.

(i) Total surface area of the herbarium

$= 2(lb + bh + hl)$

$= 2(30 \times 25 + 25 \times 25 + 25 \times 30)$ cm^2

$= 2(750 + 625 + 750)$ cm^2

$= 2 \times 2125$ cm^2 = 4250 cm^2

Hence, area of the glass = 4250 cm^2

(ii) A cuboid has 12 edges. These consist of 4 lengths, 4 breadths and 4 heights.

∴ length of the tape required

$= 4l + 4b + 4h$

$= (4 \times 30 + 4 \times 25 + 4 \times 25)$ cm

$= (120 + 100 + 100)$ cm = 320 cm

Q.7. *Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions* 25 *cm* × 20 *cm* × 5 *cm and the smaller of dimesnsions* 15 *cm* × 12 *cm* × 5 *cm. For all the overlaps,* 5% *of the total surface area is required extra. If the cost of the cardboard is Rs* 4 *for* 1000 *cm*^{2}*, find the cost of cardboard required for supplying* 250 *boxes of each kind.*

Sol. For bigger boxes :

l = 25 cm, b = 20 cm, h = 5 cm

Total surface area of 1 bigger box

$= 2(lb + bh + hl)$

$= 2(25 \times 20 + 20 \times 5 + 5 \times 25)$ cm^2

$= 2\,(500 + 100 + 125)$ cm^2 = 1450 cm^2

Area of cardboard required for overlaps

= 5% of 1450 cm^2 = $\dfrac{1450 \times 5}{100}$ cm^2 = 72.5 cm^2

Total area of cardboard needed for

1 bigger box $= (1450 + 72.5)\ cm^2$

$= 1522.5\ cm^2$

Total area of cardboard needed for 250 bigger boxes $= 1522.5 \times 250\ cm^2 = 380625\ cm^2$

For smaller boxes :

$l = 15$ cm, $b = 12$ cm, $h = 5$ cm

Total surface area of 1 smaller box

$= 2\ (lb + bh + hl)$

$= 2(15 \times 12 + 12 \times 5 + 5 \times 15)\ cm^2$

$= 2\ (180 + 60 + 75)\ cm^2 = 630\ cm^2$

Area of cardboard required for overlaps

$= 5\%$ of $630\ cm^2$

$= \frac{630 \times 5}{100}\ cm^2 = 31.5\ cm^2$

Total area of cardboard needed for

1 smaller box $= (630 + 31.5)\ cm^2 = 661.5\ cm^2$

Total area of cardboard needed for 250 smaller boxes $= 661.5 \times 250\ cm^2 = 165375\ cm^2$

Now, total area of cardboard needed for 500 boxes (250 bigger and 250 smaller boxes)

$= (380625 + 165375)\ cm^2 = 546000\ cm^2$

Cost of $1000\ cm^2$ of cardboard = Rs 4

$\therefore$ Cost of $546000\ cm^2$ of cardboard

$= \text{Rs } \frac{4}{1000} \times 546000 = \text{Rs } 2184$

Q.8. *Parveen wanted to make a temporary shelter for her car, by making a box-like structuer with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimesions 4 m × 3 m?*

Sol. Here, $l = 4$ m, $b = 3$ m, $h = 2.5$ m

The tarpaulin is needed to cover 5 faces only (excluding the floor)

Surface area of the shelter

$= lb + 2\ (bh + hl)$

$= 4 \times 3\ m^2 + 2(3 \times 2.5 + 2.5 \times 4)\ m^2$

$= 12\ m^2 + 2(7.5 + 10)\ m^2$

$= (12 + 35)\ m^2 = 47\ m^2$

Hence, $47\ m^2$ of tarpaulin is required to make the shelter.

OTHER IMPORTANT QUESTIONS

Q.1. *A cuboid has 12 edges. The combined length of all 12 edges of the cuboid is equal to* :

(*a*) *length + breadth + height* **[Imp.]**

(*b*) *4 × length × breadth × height*

(*c*) *4 × (length + breadth + height)*

(*d*) *3 × (length + breadth + height)*

Sol. (*c*) Let the length, breadth and height of the cuboid be x, y and z respectively.

Then, combined length of all 12 edges of the cuboid $= 2(x + y) + 2\ (y + z) + 2(z + x) = 4\ (x + y + z)$.

Q.2. *Length, breadth and height of a cuboid are l, b and h respectively. Length of the diagonal of the cuboid is* : **[2010]**

(*a*) $l^2 + b^2$ (*b*) $l^2 + b^2 + h^2$

(*c*) $\sqrt{l^2 + b^2 + h^2}$ (*d*) $\sqrt{l^2 + b^2}$

Sol. (*c*) The length of the diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$

Q.3. *The dimensions of a box are 1m, 80 cm and 50 cm. The area of its four walls is :*

(*a*) $1200\ cm^2$ (*b*) $15000\ cm^2$

(*c*) $18000\ cm^2$ (*d*) $20000\ cm^2$

Sol. (*c*) Required area $= 2h\ (l + b)$

$= 2 \times 50\ (100 + 80)\ cm^2$

$= 100 \times 180\ cm^2 = 18000\ cm^2$

Q.4. *Five cubes each of edge 1 cm are joined face to face. The surface area of the cuboid thus formed is :* **[V. Imp.]**

(*a*) $5\ cm^2$ (*b*) $10\ cm^2$

(*c*) $11\ cm^2$ (*d*) $22\ cm^2$

Sol. (*d*) Length of the cuboid thus formed $= 5$ cm

Surface area of the cuboid

$= 2(5 \times 1 + 1 \times 1 + 1 \times 5)\ cm^2$

$= 22\ cm^2$

Q.5. *Maximum length of rod that can be kept in a cuboidal box of sides* 30 *cm,* 24 *cm and* 18 *cm is* : **[2010]**

(a) $30\sqrt{2}$ cm (b) $20\sqrt{2}$ cm

(c) $25\sqrt{2}$ cm (d) $40\sqrt{2}$ cm

Sol. (a) Maximum length of the rod that can be kept in the box = length of the diagonal of the box = $\sqrt{30^2+24^2+18^2}$ cm

$= \sqrt{900+576+324}$ cm

$= \sqrt{1800}$ cm $= 30\sqrt{2}$ cm

Q.6. *If the length of the diagonal of a cube is* $6\sqrt{3}$ *cm, find the edge of the cube.*

Sol. We have, $a\sqrt{3} = 6\sqrt{3} \Rightarrow a = 6$ cm

Hence, edge of the cube = 6 cm.

Q.7. *The perimeter of one face of a cube is* 20 *cm. Find its surface area.* **[2010]**

Sol. Perimeter of one face = 20 cm

$\therefore$ Edge of the cube $= \frac{20}{4}$ cm = 5 cm.

$\therefore$ Surface area of the cube $= 6 \times 5^2$ cm^2

$= 150$ cm^2

Q.8. *The length, breadth and height of a room are* 5 *m,* 4 *m and* 3 *m respectively. Find the cost of painting the walls of the room and the ceiling at the rate of Rs* 50 *per* m^2 **[2010]**

Sol. Area of four walls of a room = lateral surface area of the room

$= 2h\ (l + b) = 2 \times 3(5 + 4)$ m^2 = 54 m^2.

Area of the ceiling

$= l \times b = 5 \times 4$ m^2 = 20 m^2

$\therefore$ Total area to be painted

$= (54 + 20)$ m^2 = 74 m^2

Cost of painting = Rs 50×74 = Rs 3700

Q.9. *The dimensions of a cuboid are in the ratio* 3 : 4 : 5 *and its total surface area is* 3384 *cm*^{2}*. Find the dimensions of the solid.* **[2010, 2011 (T-II)]**

Sol. Let the dimensions of the cuboid be $3x$, $4x$ and $5x$.

Then total surface area of the cuboid

$= 2(3x \times 4x + 4x \times 5x + 5x \times 3x)$

$= 2(12x^2 + 20x^2 + 15x^2) = 94x^2$ cm^2

$\therefore 94x^2 = 3384$ (Given)

$\Rightarrow x^2 = \frac{3384}{94} = 36$

$\Rightarrow x = 6$

$\therefore$ Dimensions of the solid are 3×6 cm, 4×6 cm and 5×6 cm or 18 cm, 24 cm and 30 cm.

Q.10. *Sumit has built a cubical water tank with lid for his house, with each outer edge* 1.5 *m long. He gets the outer surface of the tank excluding the base covered with square tiles of side* 25 *cm. Find how much he would spend for the tiles, if the cost of tiles is Rs* 360 *per dozen.* **[2010, 2011 (T-II)]**

Sol. The tank has 6 square faces, out of which 5 (excluding the base) are to be covered with square tiles.

$\therefore$ Area of the 5 faces to be covered with square tiles $= 5 \times 1.5^2$ m^2

Area of 1 tile $= 0.25 \times 0.25$ m^2

$\therefore$ Number of square tiles needed to cover the 5 surfaces of the tank $= \frac{5 \times 1.5^2}{0.25 \times 0.25} = 180$

Cost of 12 squares tiles = Rs 360

$\therefore$ Cost of 180 tiles = Rs $\frac{360}{12} \times 180$ = Rs 5400

PRACTICE EXERCISE 13.1A

Choose the correct option (Q 1 – 6) :

1 Mark Questions

1. The length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5 m), is :

(a) 15 m (b) 16 m

(c) 10 m (d) 12 m

2. The lateral surface area of a cube is 100 cm^2. Its total surface area is :

(a) 50 cm^2 (b) 125 cm^2

(c) 140 cm^2 (d) 150 cm^2

3. The dimensions of a room are 4 m × 3 m × 2 m. The area of the four walls of the room is :

(a) 28 m^2 (b) 56 m^2

(c) 60 m^2 (d) 70 m^2

4. Two cubes each of edge 5 cm are joined face to face. The surface area of the cuboid thus formed is equal to :

(*a*) 200 cm^2 (*b*) 250 cm^2

(*c*) 360 cm^2 (*d*) 280 cm^2

5. A cuboid of dimensions 10 cm × 5 cm × 5 cm is cut into two cubes, each of edge 5 cm. Ratio of the total surface area of the cuboid to the total surface area of the two cubes is : **[HOTS]**

(*a*) 5 : 6 (*b*) 6 : 5

(*c*) 1 : 1 (*d*) 2 : 3

6. The length and breadth of a room are 4 m and 2 m respectively. If the areas of the four walls of the room is 24 m^2, then the height of the room is :

(*a*) 1 m (*b*) 1.5 m

(*c*) 2 m (*d*) 2.5 m

2 Marks Questions

7. Find the length of the diagonal of a cuboid of dimensions 7 cm × 5 cm × 5 cm.

8. If the length of the diagonal of a cube is $4\sqrt{3}$ cm, find the edge of the cube.

9. Three cubes each of side 6 cm are joined end to end. Find the surface area of the resulting cuboid. **[2011 (T-II)]**

3 Marks Questions

10. Three cubes are placed adjacent to each other in a row. Find the ratio of the total surface area of the cuboid thus formed to the sum of the surface areas of the three cubes. **[V. Imp.]**

11. The dimensions of a cuboid are in the ratio 3 : 2 : 1. If the lateral surface area of the cuboid is 360 cm^2, find its total surface area.

12. 10 cubes are placed adjacent to each other in a row. Find the ratio of the total surface area of the cuboid thus formed to the sum of the surface areas of the 10 cubes.

13. The dimensions of a cuboid are in the ratio 5 : 3 : 2. If the total surface area of the cuboid is 248 cm^2, find the dimensions of the cuboid. **[Imp.]**

13.2 SURFACE AREA OF A RIGHT CIRCULAR CYLINDER

1. Curved surface area of a cylinder of base radius r and height h is $2\pi rh$.
2. Total surface area of a cylinder of base radius r and height h is $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$

TEXTBOOK'S EXERCISE 13.2

Q.1. *The curved surface area of a right circular cylinder of height* 14 *cm is* 88 cm^2. *Find the diameter of the base of the cylinder.* **[2011 (T-II)]**

Sol. Here, h = 14 cm, curved surface area = 88 cm^2, r = ?

Curved surface area of the cylinder = $2\pi rh$

$\Rightarrow 88 = 2 \times \frac{22}{7} \times r \times 14$

$\Rightarrow 88 = 44 \times 2 \times r \Rightarrow r = \frac{88}{44 \times 2} = 1$

Hence, base diameter of the cylinder

= 1 × 2 cm = 2 cm

Q.2. *It is required to make a closed cylindrical tank of height* 1 *m and base diameter* 140 *cm from a metal sheet. How many square metres of the sheet are required for the same?*

Sol. Here, h = 1 m, $r = \frac{140}{2}$ cm

= 70 cm = 0.7 m

Total surface area of the cylinder

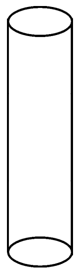
$= 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 0.7\,(1 + 0.7)\ m^2$

$= 44 \times 0.1 \times 1.7\ m^2 = 7.48\ m^2$

Hence, 7.48 m^2 of sheet is required

Q.3. *A metal pipe is* 77 *cm long. The inner diameter of a cross section is* 4 *cm, the outer diameter being* 4.4 *cm (see figure). Find its.*



(i) inner curved surface area,

(ii) outer curved surface area,

(iii) total surface area.

Sol. Here, $h = 77$ cm,

Outer radius (R) = $\frac{4.4}{2}$ cm = 2.2 cm

Inner radius (r) = $\frac{4}{2}$ cm = 2 cm

(i) Inner curved surface area of the pipe = $2\pi rh = 2 \times \frac{22}{7} \times 2 \times 77\ \text{cm}^2$

$= 2 \times 22 \times 22\ \text{cm}^2 = 968\ \text{cm}^2$

(ii) Outer curved surface area of the pipe

$= 2\pi Rh = 2 \times \frac{22}{7} \times 2.2 \times 77\ \text{cm}^2$

$= 44 \times 24.2\ \text{cm}^2$

$= 1064.80\ \text{cm}^2$

(iii) Total surface area of the pipe = inner curved surface area + outer curved surface area + areas of the two base rings.

$= 2\pi rh + 2\pi Rh + 2\pi\ (R^2 - r^2)$

$= 968\ \text{cm}^2 + 1064.80\ \text{cm}^2$

$+ 2 \times \frac{22}{7}\ [(2.2)^2 - 2^2]\ \text{cm}^2$

$= 2032.80\ \text{cm}^2 + 5.28\ \text{cm}^2 = 2038.08\ \text{cm}^2$

Q.4. *The diameter of a roller is* 84 *cm and its length is* 120 *cm. It takes* 500 *complete revolutions to move once over to level a playground. Find the area of the playground in* m^2. **[2011 (T-II)]**

Sol. Radius of the roller (r) = $\frac{84}{2}$ cm = 42 cm

Length of the roller (h) = 120 cm

Curved surface area of the roller

$= 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120\ \text{cm}^2$

$= 44 \times 720\ \text{cm}^2 = 31680\ \text{cm}^2$

$\therefore$ Area covered by the roller in 1 revolution

$= 31680\ \text{cm}^2$

$\therefore$ Area covered by the roller in 500 revolutions = $31680 \times 500\ \text{cm}^2 = 15840000\ \text{cm}^2$

Hence, area of the playground

$= \frac{15840000}{100 \times 100}\ \text{m}^2 = 1584\ \text{m}^2$

Q.5. *A cylindrical pillar is* 50 *cm in diameter and* 3.5 *m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs* 12.50 *per* m^2.

Sol. Here, $r = \frac{50}{2}$ cm = 25 cm = 0.25 m, $h = 3.5$ m

Curved surface area of the pillar

$= 2\pi rh = 2 \times \frac{22}{7} \times 0.25 \times 3.5\ \text{m}^2 = 5.5\ \text{m}^2$

Cost of painting 1 m^2 = Rs 12.50

$\therefore$ Total cost of painting the curved surface of the pillar = Rs 12.50×5.5 = Rs 68.75

Q.6. *Curved surface area of a right circular cylinder is* 4.4 m^2. *If the radius of the base of the cylinder is* 0.7 *m, find its height.*

Sol. Curved surface area of the cylinder

$= 4.4\ \text{m}^2,\ r = 0.7\ \text{m},\ h = ?$

Curved surface area of the cylinder = $2\pi rh$

$\Rightarrow\ 4.4 = 2 \times \frac{22}{7} \times 0.7 \times h \Rightarrow\ h = \frac{4.4}{4.4} = 1$

Hence, height of the cylinder is 1 m

Q.7. *The inner diameter of a circular well is* 3.5 *m. It is* 10 *m deep. Find*

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of Rs 40 *per* m^2.

Sol. Here, $r = \frac{3.5\ \text{m}}{2}$, $h = 10$ m

(i) Inner curved surface area of the well

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10\ \text{m}^2$

$= 22 \times 5\ \text{m}^2 = 110\ \text{m}^2$

(ii) Cost of plastering 1 m^2 = Rs 40

$\therefore$ Cost of plastering the curved surface area of the well = Rs 110×40 = Rs 4400

Q.8. *In a hot water heating system. there is a cylindrical pipe of length* 28 *m and diameter* 5 *cm. Find the total radiating surface in the system.* **[2011 (T-II)]**

Sol. Here, $r = \frac{5}{2}$ cm = 2.5 cm = 0.025 m, h = 28 m.

Total radiating surface in the system
= total surface area of the cylinder

$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 0.025\,(28 + 0.025)\text{ m}^2$

$= \frac{44 \times 0.025 \times 28.025}{7}\text{ m}^2 = 4.4\text{ m}^2$ (approx)

Q.9. *Find (i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is* 4.2 *m in diameter and* 4.5 *m high.*

(ii) how much steel was actually used, if $\frac{1}{12}$ *of the steel actually used was wasted in making the tank.*

Sol. Here, $r = \frac{4.2}{2}$ m = 2.1 m, h = 4.5 m

(i) Curved surface area of the storage tank

$= 2\pi rh = 2 \times \frac{22}{7} \times 2.1 \times 4.5\text{ m}^2 = 59.4\text{ m}^2$

(ii) Total surface area of the tank $= 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 2.1\,(4.5 + 2.1)\text{ m}^2$

$= 44 \times 0.3 \times 6.6\text{ m}^2 = 87.12\text{ m}^2$

Let the actual area of steel used be x m^2.

Area of steel wasted $= \frac{1}{12}$ of $x\text{ m}^2 = \frac{x}{12}\text{ m}^2$ .... (i)

$\therefore$ Area of the steel used in the tank

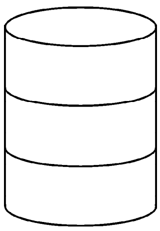
$$= \left(x - \frac{x}{12}\right)\text{m}^2 = \frac{11}{12}\,x\text{ m}^2$$

$\Rightarrow 87.12 = \frac{11}{12}x$

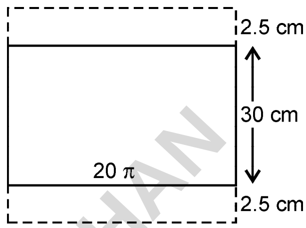
$\Rightarrow x = \frac{87.12 \times 12}{11} = 95.04\text{ m}^2$

Hence, 95.04 m^2 of steel was actually used.

Q.10. *In the figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of* 20 *cm and height of* 30 *cm. A margin of* 2.5 *cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.* **[Imp.]**



Sol. Here, $r = \frac{20}{2}$ cm = 10 cm, Height = 30 cm



Circumference of the base of the frame = $2\pi r$
$= 2\pi \times 10$ cm $= 20\pi$ cm

Height of the frame = 30 cm

Height of the cloth needed for covering the frame (including the margin)
= (30 + 2.5 + 2.5) cm = 35 cm

Also, breadth of the cloth
= circumference of the base of the frame.

$\therefore$ Area of the cloth required for covering the lampshade = length × breadth

$= 35 \times 20\pi\text{ cm}^2$

$= 35 \times 20 \times \frac{22}{7}\text{ cm}^2 = 2200\text{ cm}^2$

Q.11. *The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius* 3 *cm and height* 10.5 *cm. The Vidyalaya was to supply the competitors with cardboard. If there were* 35 *competitors, how much cardboard was required to be bought for the competition?* **[2011 (T-II)]**

Sol. Here, r = 3 cm, h = 10.5 cm

The penholders have only one base i.e., these are open at one end.

Total surface area of 1 penholder

$= 2\pi rh + \pi r^2 = \pi r\,(2h + r)$

$= \frac{22}{7} \times 3\,(2 \times 10.5 + 3)\text{ cm}^2$

$= \frac{22}{7} \times 3 \times 24\text{ cm}^2$

Total surface area of 35 penholders

$= \frac{22}{7} \times 3 \times 24 \times 35\text{ cm}^2 = 7920\text{ cm}^2$

Hence, 7920 cm^2 of cardboard is needed.

OTHER IMPORTANT QUESTIONS

Q.1. *The radii of two right circular cylinders are in the ratio of* 4 : 5 *and their heights are in the ratio* 2 : 3. *The ratio of their curved surface areas is equal to :*

(*a*) 8 : 15 (*b*) 36 : 81
(*c*) 2 : 3 (*d*) 16 : 25

Sol. (*a*) Required ratio

$$= \frac{2\pi \times (4x) \times (2y)}{2\pi \times (5x) \times (3y)} = \frac{8}{15} = 8 : 15$$

Q.2. *The radius and height of a cylindrical box, without lid, are r and h respectively. The total outer surface area of the box is* : **[Imp.]**

(*a*) $\pi h (2r + h)$ (*b*) $\pi r (h + 2r)$
(*c*) $\pi r (2h + r)$ (*d*) $\pi (2h + r)$

Sol. (*c*) Required area

$$= 2\pi rh + \pi r^2 = \pi r (2h + r).$$

Q.3. *A square piece of paper of side* 12 *cm is rolled to form a cylinder. The curved surface of the cylinder will be* :

(*a*) $12\pi\ cm^2$ (*b*) $144\ cm^2$
(*c*) $12\ cm^2$ (*d*) $24\ cm^2$

Sol. (*b*) Height (h) of the cylinder = 12 cm.
Circumference of the base of the cylinder = 12 cm **[2010, 2011 (T-II)]**

$\Rightarrow 2\pi r = 12$ cm
$\therefore$ Curved surface area of the cylinder
$= 2\pi rh = 12 \times 12\ \text{cm}^2 = 144\ \text{cm}^2$

Q.4. *A rectangular sheet of paper* 22 *cm* × 15 *cm is rolled along its length to form a hollow cylinder. The radius of the cylinder is* **[2011 (T-II)]**

(*a*) 7 *cm* (*b*) 15 *cm*
(*c*) 11 *cm* (*d*) 3.5 *cm*

Sol. (*d*) In this case, circumference of the base of the cylinder = 22 cm

$$\Rightarrow 2\pi r = 22 \Rightarrow \frac{2 \times 22}{7} \times r = 22$$

$$\Rightarrow r = \frac{7}{2}\text{cm} = 3.5 \text{ cm}$$

Q.5. *The curved surface area of a right circular cylinder of height* 14 *cm is* 88 *cm*^{2}. *Diameter of the cylinder is* : **[2010]**

(*a*) 1 *cm* (*b*) 2 *cm*
(*c*) 3 *cm* (*d*) 3 *cm*

Sol. (*b*) We have, $2\pi rh = 88$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88$$

$$\Rightarrow r = \frac{88}{2 \times 22 \times 2} = 1$$

$\therefore$ Diameter of the cylinder = $2r$ = 2 cm

Q.6. *The curved surface area of a cylinder is* 4400 *cm*^{2} *and the circumference of its base is* 110 *cm Find the height of the cylinder.* **[2010, 2011 (T-II)]**

Sol. We have $2\pi r = 110$ cm
Also, curved surface area of the cylinder
$= 4400\ \text{cm}^2$
$\Rightarrow 2\pi rh = 4400$

$$\Rightarrow 110 \times h = 4400 \Rightarrow h = \frac{4400}{110} = 40 \text{ cm}$$

Hence, height of the cylinder = 40 cm

Q.7. *Find the lateral surface area of a solid cylinder having diameter* 50 *cm and height* 3.5 *m.* **[2010]**

Sol. We have,

$$r = \frac{50}{2} \text{ cm} = 0.25 \text{ m}, \quad h = 3.5 \text{ m}$$

$\therefore$ Lateral surface area of the cylinder

$$= 2\pi rh = 2 \times \frac{22}{7} \times 0.25 \times 3.5\ \text{m}^2 = 5.5\ \text{m}^2$$

Q.8. *The diameter of a roller,* 120 *cm long is* 84 *cm. It takes* 500 *complete revolutions to level a playground. Find the cost of levelling it at the rate of Rs* 25 *per sq metre.* **[2010, 2011 (T-II)]**

Sol. Length of roller (h) = 120 cm
Radius of the roller (r) = 42 cm
Curved surface area of the roller

$$= 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120\ \text{cm}^2$$

But, area covered by the roller in 1 revolution = curved surface area of the roller

$$= 2 \times \frac{22}{7} \times 42 \times 120\ \text{cm}^2$$

$\therefore$ Area covered by the roller in 500 revolutions $= 2 \times \frac{22}{7} \times 42 \times 120 \times 500$ cm^2

$= 15840000$ cm^2 $= 1584$ m^2

$\therefore$ Cost of levelling the ground

$=$ Rs $25 \times 1584 =$ Rs 39,600

Q.9. *A roller* 1.5 *m long has a diameter of* 70 *cm. How many revolutions will it make to level a playground measuring* 50 *m* × 33 *m?*

[2010, 2011 (T-II)]

Sol. Length of roller (h) = 1.5 m

Radius of the roller (r) $= \frac{70}{2}$ cm $= 0.35$ m

Area of the playground $= 50 \times 33$ m^2

Area covered by the roller in 1 revolution

= curved surface area of the roller

$= 2 \times \frac{22}{7} \times 0.35 \times 1.5$ m^2 $= 3.3$ m^2.

$\therefore$ Number of revolutions made by the roller to level the playground

$$= \frac{\text{area of the playground}}{\text{curved surface area of the roller}} = \frac{50 \times 33}{3.3}$$

$= 500$

Q.10. *The inner diameter of a circular well is* 10 *m. It is* 12 *m deep. Find the cost of plastering the curved surface at the rate of Rs* 60 *per m*^{2}. *(Use* $\pi = 3.14$ *approx)*

[2010, 2011 (T-II)]

Sol. Radius of the well (r) = 5 m

Depth of the well (h) = 12 m

$\therefore$ Curved surface area of the well

$= 2\pi rh = 2 \times 3.14 \times 5 \times 12$ m^2

$\therefore$ Cost of plastering the inner surface of the well = Rs $(2 \times 3.14 \times 5 \times 12 \times 60)$ = Rs 22,608

PRACTICE EXERCISE 13.2A

Choose the correct option (Q 1 – 6) :

1 Mark Questions

1. Curved surface area of a right circular cylinder is 8.8 m^2. If the radius of the base of the cylinder is 1.4 m, its height is equal to : **[2011 (T-II)]**

(*a*) 10 m (*b*) 100 m

(*c*) 0.1 m (*d*) 1 m

2. The total surface area of a cylinder of base diameter r and height $2r$ is :

(*a*) $\pi r(h + r)$ (*b*) $5\pi r^2$

(*c*) $\frac{5}{2}\pi r^2$ (*d*) $\frac{2}{5}\pi r^2$

3. A square piece of side 10 cm is rolled to form a cylinder. The curved surface area of the cylinder is : **[Imp.]**

(*a*) 10π cm^2 (*b*) 100π cm^2

(*c*) 100 cm^2 (*d*) 150 cm^2

4. In a cylinder, if radius is doubled and height is halved, its curved surface area will be :

(*a*) halved (*b*) doubled

(*c*) same (*d*) four times

5. The base radii of two cylinders are in the ratio 1 : 2 and their heights are in the ratio 3 : 2. The ratio of their curved surface areas is :

(*a*) 1 : 4 (*b*) 3 : 4

(*c*) 4 : 3 (*d*) 4 : 1

6. The total surface area of a cylinder of radius 5 cm and height 16 cm is :

(*a*) 600 cm^2 (*b*) $660\ \pi$ cm^2

(*c*) 660 cm^2 (*d*) 700 cm^2

2 Marks Questions

7. Check whether the following statement is true or not :

If the radius of a right circular cylinder is halved and height is doubled, the total surface area will remain unchanged.

8. A rectangular strip 5 cm × 23 cm is rotated completely about the 23 cm side. Find the total surface area of the cylinder so formed. **[HOTS]**

9. Find the curved surface area ofa cylindrical pillar which is 1.2 m high and has the diameter of the base as 28 cm. [Use π $\frac{22}{7}$] **[2011 (T-II)]**

10. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height. [Use $\pi = \frac{22}{7}$] **[2011 (T-II)]**

3 Marks Questions

11. The radius of a roller, 1.2 m long, is 0.42 m. If it takes 200 complete revolutions to level a playground, find the area of the playground. **[2010]**

12. A school provides milk to the students daily in cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto a height of 12 cm, find how many litres of milk is needed to serve 1600 students.[Use $\pi \frac{22}{7}$] **[2011 (T-II)]**

13.3 SURFACE AREA OF A RIGHT CIRCULAR CONE

1. Curved surface area of a cone of base radius r and slant height l is πrl.

2. $l^2 = r^2 + h^2$, where l, r and h are slant height, base radius and height of the cone.

3. Total surface area of a cone of base radius, r and slant height l is $\pi r(l + r)$.

TEXTBOOK'S EXERCISE 13.3

Q.1. *Diameter of the base of a cone is* 10.5 *cm and its slant height is* 10 *cm. find its curved surface area.*

Sol. Here, $r = \frac{10.5}{2}$ cm = 5.25 cm, l = 10 cm.

Curved surface area of the cone = πrl

$= \frac{22}{7} \times 5.25 \times 10 \text{ cm}^2 = 165 \text{ cm}^2$

Q.2. *Find the total surface area of a cone, if its slant height is* 21 *m and diameter of its base is* 24 *m.* **[2011 (T-II)]**

Sol. Here, l = 21 m, $r = \frac{24}{2}$ m = 12 m

Total surface area of the cone

$= \pi r(l + r) = \frac{22}{7} \times 12\,(21 + 12) \text{ m}^2$

$= \frac{22}{7} \times 12 \times 33 \text{ m}^2 = 1244.57 \text{ m}^2$

Q.3. *Curved surface area of a cone is* 308 *cm*^{2} *and its slant height is* 14 *cm. Find (i) radius of the base and (ii) total surface area of the cone.* **[2010, 2011 (T-II)]**

Sol. Here, l = 14 cm, curved surface area = 308 cm^2, r = ?

(i) Curved surface area of the cone = πrl

$\Rightarrow 308 = \frac{22}{7} \times r \times 14 \Rightarrow r = \frac{308}{22 \times 2} = 7$

Hence, base radius of the cone = 7 cm.

(ii) Total surface area of the cone $= \pi r\,(l + r)$

$= \frac{22}{7} \times 7\,(14 + 7) \text{ cm}^2 = 22 \times 21 \text{ cm}^2 = 462 \text{ cm}^2$

Q.4. *A conical tent is* 10 *m high and the radius of its base is* 24 *m. Find*

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 *m*^{2} *canvas is Rs* 70.

[2010, 2011 (T-II)]

Sol. Here, h = 10 m, r = 24 m

(i) We have, $l^2 = h^2 + r^2 = (10)^2 + (24)^2 = 100 + 576 = 676$

$\Rightarrow l = \sqrt{676}$ m = 26 m

(ii) Curved surface area of the tent

$= \pi rl = \frac{22}{7} \times 24 \times 26 \text{ m}^2$

Cost of 1 m^2 canvas = Rs 70

$\therefore$ Cost of $\frac{22}{7} \times 24 \times 26$ m^2 of canvas

$= \text{Rs } 70 \times \frac{22}{7} \times 24 \times 26 = \text{Rs } 137280$

Q.5. *What length of tarpaulin* 3 *m wide will be required to make a conical tent of height* 8 *m and base radius* 6 *m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately* 20 *cm. (use* π = 3.14) **[2011 (T-II)]**

Sol. Here $h = 8$ m, $r = 6$ m

We have,

$l = \sqrt{r^2 + h^2} = \sqrt{36 + 64}$ m $= \sqrt{100}$ m $= 10$ m

Curved surface area of the tent

$= \pi rl = 3.14 \times 6 \times 10$ m^2

$\therefore$ Required length of tarpaulin

$= \dfrac{3.14 \times 6 \times 10}{3}$ m + 20 cm

= 62.8 m + 0.2 m = 63 m

Q.6. *The slant height and base diameter of a conical tomb are* 25 *m and* 14 *m respectively. Find the cost of whitewashing its curved surface at the rate of Rs* 210 *per* 100 *m*^{2}.

Sol. Here, $l = 25$ m, $r = \dfrac{14}{2}$ m = 7 m

Curved surface area of the tomb $= \pi rl$

$= \dfrac{22}{7} \times 7 \times 25$ m^2 = 550 m^2

Cost of whitewashing 100 m^2 = Rs 210

$\therefore$ Cost of whitewashing 550 m^2

$=$ Rs $\dfrac{210}{100} \times 550 =$ Rs 1155

Q.7. *A joker's cap is in the form of a right circular cone of base radius* 7 *cm and height* 24 *cm. Find the area of the sheet required to make* 10 *such caps.* **[2011 (T-II)]**

Sol. Here, $r = 7$ cm, $h = 24$ cm

We have, $l = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + 7^2}$ m

$= \sqrt{576 + 49}$ m $= \sqrt{625}$ m = 25 cm

Total curved surface area of 1 cap $= \pi rl$

$= \dfrac{22}{7} \times 7 \times 25$ cm^2 = 550 cm^2

Area of sheet required to make 10 such caps

$= 10 \times 550$ cm^2 = 5500 cm^2

Q.8. *A bus stop is barricaded from the remaining part of the road, by using* 50 *hollow cones made of recycled cardboard. Each cone has a base diameter of* 40 *cm and height* 1 *m. If the outer side of each of the cones is to be painted and the cost of painting is Rs* 12 *per m*^{2}*, what will be the cost of painting all these cones?*

(Use $\pi = 3.14$ *and take* $\sqrt{1.04} = 1.02$)

Sol. Here,

$r = \dfrac{40}{2}$ cm = 20 cm = 0.20 m, $h = 1$ m

$l = \sqrt{h^2 + r^2}$ m $= \sqrt{1^2 + (0.2)^2}$ m $= \sqrt{1.04}$ m

= 1.02 m

Curved surface area of 1 cone $= \pi rl$

Curved surface area of 50 cones

$= 50 \times 3.14 \times 0.2 \times 1.02$ m^2 = 32.028 m^2

Cost of painting an area of 1 m^2 = Rs 12

$\therefore$ Cost of painting an area of 32.028 m^2

= Rs 12 × 32.028 = Rs 384.34 (approx)

OTHER IMPORTANT QUESTIONS

Q.1. *The diameters of two cones are equal. If their slant heights are in the ratio* 7 : 8*, then the ratio of their curved surface areas will be :* **[Imp.]**

(*a*) 2 : 3 (*b*) 5 : 6

(*c*) 4 : 5 (*d*) 7 : 8

Sol. (*d*) Required ratio $= \dfrac{\pi r \times 7x}{\pi r \times 8x} = 7 : 8$

Q.2. *The height of a cone is equal to its base diameter. The slant height of the cone is :* **[V. Imp.]**

(*a*) $\sqrt{r^2 + h^2}$ (*b*) $r\sqrt{5}$

(*c*) $h\sqrt{5}$ (*d*) $rh\sqrt{5}$

Sol. (*b*) Slant height

$= \sqrt{r^2 + h^2} = \sqrt{r^2 + 4r^2} = r\sqrt{5}$. [$\because h = 2r$]

Q.3. *A joker's cap is in the form of a right circular cone of base radius* 7 *cm and height* 24 *cm. The area of the sheet required to make* 10 *such caps is :*

(*a*) 2500 *cm*^{2} (*b*) 3500 *cm*^{2}

(*c*) 4500 *cm*^{2} (*d*) 5500 *cm*^{2}

Sol. (*d*) Slant height of the cap

$= \sqrt{7^2 + 24^2}$ cm $= \sqrt{49 + 576}$ cm = 25 cm

$\therefore$ Required area $= 10 \times \dfrac{22}{7} \times 7 \times 25$ cm^2

$= 5500$ cm^2.

Q.4. *The circumference of the base of a right circular cone is 44 cm and its slant height is 10 cm. Its curved surface area is :* **[2010]**

(*a*) $\dfrac{220}{7}$ cm^2 (*b*) $\dfrac{200}{7}$ cm^2

(*c*) 200 cm^2 (*d*) 220 cm^2

Sol. (*d*) We have, $2\pi r = 44$

$$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$\therefore$ Curved surface area of the cone

$$= \pi rl = \frac{22}{7} \times 7 \times 10 \text{ cm}^2 = 220 \text{ cm}^2$$

Q.5. *The curved surface area of a cone of radius 8 cm is 352 cm^2. Find its height.*

Sol. We have, $\pi rl = 352$

$$\Rightarrow \frac{22}{7} \times 8 \times l = 352 \Rightarrow l = \frac{352 \times 7}{22 \times 8} = 14 \text{ cm}$$

$$\therefore \; h = \sqrt{l^2 - r^2} = \sqrt{14^2 - 8^2} \text{ cm}$$

$$= \sqrt{132} \text{ cm} = 2\sqrt{33} \text{ cm.}$$

Q.6. *If slant height of a cone is 21 m and diameter of its base is 24 m, then find its total surface area.* **[2010, 2011 (T-II)]**

Sol. We have, $l = 21$ m, $r = \dfrac{24}{2}$ m $= 12$ m

$\therefore$ Total surface area of the cone

$$= \pi r(l + r) = \frac{22}{7} \times 12\ (21 + 12) \text{ m}^2$$

$$= \frac{22}{7} \times 12 \times 33 \text{ m}^2 = 1244\frac{4}{7} \text{ m}^2$$

Q.7. *How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?* **[2010, 2011 (T-II)]**

Sol. Slant height of the cone

$$= \sqrt{3.5^2 + 12^2} \text{ m} = \sqrt{156.25} \text{ m} = 12.5 \text{ m}$$

$\therefore$ Area of canvas required to make the tent

$$= \pi rl = \frac{22}{7} \times 12 \times 12.5 \text{ m}^2 = 471.42 \text{ m}^2.$$

Q.8. *The height of a conical tent is 7 m and the radius of its base is 24 m. What lengths of cloth of width 100 cm is needed to make the tent?* **[2010]**

Sol. We have, $h = 7$ m, $r = 24$ m

$$\therefore \; l = \sqrt{h^2 + r^2} = \sqrt{49 + 576} \text{ m} = \sqrt{625} \text{ m}$$

$= 25$ m

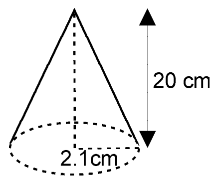
Curved surface area of the tent

$$= \pi rl = \frac{22}{7} \times 24 \times 25 \text{ m}^2$$

$\therefore$ Length of cloth required to make the tent

$$= \frac{\frac{22}{7} \times 24 \times 25}{1} \text{ m} = 1885.7 \text{ m}$$

Q.9. *A corn cob, shaped like cone has the radius of the base as 2.1 cm and height as 20 cm. If each 1 sq cm of the surface of cob carries an average of 4 grains, find how many grains you would find in the entire cob?* **[2011 (T-II)]**



Sol. Since the grains of corn are found on the curved surface of the corn cob. So, total number of grains on the corn cob

= Curved surface area of the corn cob

$\times$ Number of grains of corn on 1 cm^2.

Now, we will first find the curved surface area of the corn cob.

We have, $r = 1.2$ cm and $h = 20$ cm. Then, slant height,

$$l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (20)^2} = \sqrt{404.41}$$

$= 20.11$ cm

Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 2.1 \times 20.11 \text{ cm}^2$$

$= 132.726$ cm^2 $= 132.73$ cm^2

Hence, the total number of grains on the corn cob $= 132.73 \times 4 = 530.92$

So, there would be approximately 531 grains of corn on the cob.

PRACTICE EXERCISE 13.3A

Choose the correct option (Q 1 – 6) :

1 Mark Questions

1. The area of the iron sheet required to prepare a cone without base of height 3 cm with radius 4 cm is :

(*a*) $\frac{440}{7}$ cm^2 (*b*) 400 cm^2

(*c*) $\frac{400}{7}$ cm^2 (*d*) $\frac{7}{440}$ cm^2

2. The curved surface area of a cone of slant height $\frac{x}{2}$ is πrx. The area of its base is :

(*a*) πr^2 (*b*) $4\pi x^2$

(*c*) πx^2 (*d*) $4\pi r^2$

3. The height of a right circular cone is 24 cm and the radius of its base is 7 cm. The lateral surface area of the cone is :

(*a*) 500 cm^2 (*b*) 704 cm^2

(*c*) 550 cm^2 (*d*) 550π cm^2

4. A heap of wheat is in the form of a cone whose radius is 3 m and slant height is 7 m. The heap is to be covered by canvas to protect it from rain. The area of canvas required is :

(*a*) 50 m^2 (*b*) 60 m^2

(*c*) 66 π m^2 (*d*) 66 m^2

5. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is : **[Imp.]**

(*a*) $2\pi r\,(l + r)$ (*b*) $\pi r\left(l + \frac{r}{4}\right)$

(*c*) $\pi r\,(l + r)$ (*d*) $2\pi rl$

6. The circumference of the base of a right circular cone is 22 cm and its slant height is 8 cm. Its curved surface area is :

(*a*) 100 cm^2 (*b*) 90 cm^2

(*c*) 88 cm^2 (*d*) 77 cm^2

2 Marks Questions

7. Find the total surface area of the cone whose base radius is 8 cm and sum of base radius and slant height is 21 cm.

3 Marks Questions

8. The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone (Use π = 3.14)

9. Find the curved surface area of a cone whose base diameter is 10.5 cm and slant height is 10 cm. **[2010]**

10. How many metres of cloth 4 m wide will be required to make a conical tent whose base radius is 7 m and height 24 m? **[2010]**

11. How many metres of cloth 5 m wide will be required to make a conical tent if the radius of the base and height are 3.5 m and 12 m respectively. **[2010]**

13.4 SURFACE AREA OF A SPHERE

1. Surface area of a sphere of radius r is $4\pi r^2$.

2. Curved surface area of a hemisphere of radius r is $2\pi r^2$.

3. Total surface area of a hemisphere of radius r is $3\pi r^2$.

TEXTBOOK'S EXERCISE 13.4

Q.1. *Find the surface area of a sphere of radius :*

(i) 10.5 *cm* *(ii)* 5.6 *cm* *(iii)* 14 *cm*

Sol. (i) r = 10.5 cm

Surface area of the sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times (10.5)^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2 = 1386 \text{ cm}^2$$

(ii) $r = 5.6$ cm

Surface area of the sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times (5.6)^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 5.6 \times 5.6 \text{ cm}^2$$

$$= 394.24 \text{ cm}^2$$

(iii) $r = 14$ cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = 88 \times 28 \text{ cm}^2$$

$$= 2464 \text{ cm}^2$$

Q.2. *Find the surface area of a sphere of diameter :*

(i) 14 *cm* *(ii)* 21 *cm* *(iii)* 3.5 *m*

Sol. (i) $r = \frac{14}{2}$ cm = 7 cm

Surface area of the sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times 7^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 88 \times 7 \text{ cm}^2 = 616 \text{ cm}^2$$

(ii) $r = \frac{21}{2}$ cm = 10.5 cm

Surface area of the sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times (10.5)^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2 = 1386 \text{ cm}^2$$

(iii) $r = \frac{3.5}{2}$ m = 1.75 m

Surface area of the sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times (1.75)^2 \text{ m}^2$$

$$= 4 \times \frac{22}{7} \times 1.75 \times 1.75 \text{ m}^2 = 38.5 \text{ m}^2$$

Q.3. *Find the total surface area of a hemisphere of radius* 10 *cm. (Use* $\pi = 3.14$*)*

Sol. $r = 10$ cm

Total surface area of the hemisphere = $3\pi r^2$

$= 3 \times 3.14 \times (10)^2 \text{ cm}^2$

$= 3 \times 3.14 \times 100 \text{ cm}^2 = 942 \text{ cm}^2$

Q.4. *The radius of a spherical balloon increases from* 7 *cm to* 14 *cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.* **[2010, 2011 (T-II)]**

Sol. When $r = 7$ cm

Surface area of the balloon

$$= 4\pi r^2 = 4 \times \pi \times 7 \times 7 \text{ cm}^2$$

When $r = 14$ cm

Surface area of the balloon

$$= 4\pi r^2 = 4 \times \pi \times 14 \times 14 \text{ cm}^2$$

Required ratio of the surface areas of the balloon $= \frac{4 \times \pi \times 7 \times 7}{4 \times \pi \times 14 \times 14} = \frac{1}{4} = 1 : 4$

Q.5. *A hemispherical bowl made of brass has inner diameter* 10.5 *cm. Find the cost of tin-plating it on the inside at the rate of Rs* 16 *per* 100 *cm*^{2}. **[2011 (T-II)]**

Sol. Here $r = \frac{10.5}{2}$ cm = 5.25 cm

Inner surface area of the bowl

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2 \text{ cm}^2$$

$$= 44 \times 0.75 \times 5.25 \text{ cm}^2 = 173.25 \text{ cm}^2$$

Cost of tin plating 100 cm^2 = Rs 16

Cost of tin plating 173.25 cm^2

$$= \text{Rs } \frac{16}{100} \times 173.25 = \text{Rs } 27.72$$

Q.6. *Find the radius of a sphere whose surface area is* 154 *cm*^{2}.

Sol. Surface area of the sphere = $4\pi r^2$

$$\Rightarrow 154 = 4 \times \frac{22}{7} \times r^2 \Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{7 \times 7}{4}$$

$$\Rightarrow r = \frac{7}{2} = 3.5$$

Hence, radius of the sphere = 3.5 cm

Q.7. *The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.* **[2011 (T-II)]**

Sol. Let diameter of the earth = $2r$

Then radius of the earth = r

$\therefore$ Diameter of the moon = $\frac{2r}{4} = \frac{r}{2}$

$\therefore$ Radius of the moon = $\frac{r}{4}$

Now, surface area of the moon

$$= 4\pi\left(\frac{r}{4}\right)^2 \quad \frac{\pi r^2}{4} \qquad \text{... (i)}$$

Surface area of the earth $= 4\pi r^2$ ... (ii)

$\therefore$ Required ratio

$$= \frac{\frac{\pi r^2}{4}}{4\pi r^2} = \frac{\pi r^2}{4 \times 4\pi r^2} = \frac{1}{16} = 1 : 16$$

Q.8. *A hemispherical bowl is made of steel,* 0.25 *cm thick. The inner radius of the bowl is* 5 *cm. Find the outer curved surface area of the bowl.* **[2011 (T-II)]**

Sol. Inner radius of the bowl (r) = 5 cm

Thickness of the steel = 0.25 cm

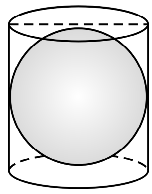
$\therefore$ Outer radius of the bowl (R)

= (5 + 0.25) cm = 5.25 cm

Outer curved surface area of the bowl = $2\pi R^2$

$$= 2 \times \frac{22}{7} \times (5.25)^2 \text{ cm}^2 = 173.25 \text{ cm}^2$$

Q.9. *A right circular cylinder just encloses a sphere of radius r (see figure). Find*



(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii). **[V. Imp]**

Sol. Here, radius of the sphere = r

Radius of the cylinder = r

And, height of the cylinder = $2r$

(i) Surface area of the sphere = $4\pi r^2$

(ii) Curved surface area of the cylinder

$= 2\pi rh = 2\pi \times r \times 2r = 4\pi r^2$

(iii) Required ratio $= \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1} = 1 : 1$

OTHER IMPORTANT QUESTIONS

Q.1. *Total surface area of a solid hemisphere of radius r is :* **[2010]**

(a) $3\pi r^2$ (b) $2\pi r^2$

(c) $4\pi r^2$ (d) $\frac{1}{3}\pi r^2$

Sol. (a) Total surface area of a hemisphere of radius $r = 2\pi r^2 + \pi r^2 = 3\pi r^2$

Q.2. *If the radius of a sphere is increased by* 5 *cm, then its surface area increases by* 704 *cm*^{2}. *The radius of the sphere before the increase was :* **[Imp.]**

(a) 2.1 *cm* (b) 3.1 *cm*

(c) 4.1 *cm* (d) 5.1 *cm*

Sol. (b) We have, $4\pi(r + 5)^2 - 4\pi r^2 = 704$

$$\Rightarrow r^2 + 25 + 10r - r^2 = \frac{704}{4\pi} = \frac{704 \times 7}{4 \times 22} = 56$$

$$\Rightarrow 10r = 56 - 25 = 31 \Rightarrow r = 3.1 \text{ cm}$$

Q.3. *A right circular cylinder of radius r just encloses a sphere. Find the surface area of the sphere.*

Sol. Radius of the cylinder

= Radius of the sphere

$\therefore$ Surface area of the sphere = $4\pi r^2$

Q.4. *Surface area of a sphere is* 154 *cm*^{2}. *Find its radius.* **[2010]**

Sol. We have $4\pi r^2 = 154$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4} \Rightarrow r = \frac{7}{2} = 3.5$$

Hence, radius of the sphere is 3.5 cm.

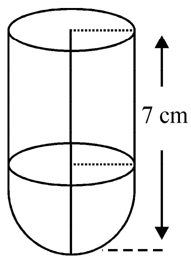
Q.5. *A cylinder of same height and radius is placed on the top of a hemisphere. Find the curved surface area of the hemisphere, if the length of the shape be 7 cm.*

Sol. Radius of the cylinder = Height of the cylinder = 3.5 cm

$\therefore$ Radius of the hemisphere = 3.5 cm

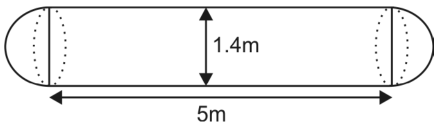
Curved surface area of the hemisphere

$= 2\ \pi \times (3.5)^2 \text{ cm}^2 = 77 \text{ cm}^2$



Q.6. *A storage tank consists of a circular cylinder, with a hemisphere adjoined on either end. If the external diameter of the cylinder be* 1.4 *m and its length be* 5 *cm, what will be the cost of painting it on the outside at the rate of Rs* 10 *per square metre?* **[2011 (T-II)]**

Sol. We have, diameter of the cylinder = 1.4 m



$\therefore$ Radius of the cylinder = $\frac{1.4}{2}$ m = 0.7 m.

Length of the cylinder = 5 m

$\therefore$ Surface area of the cylinder

$$= 2\pi rh = 2\frac{22}{7} \times 0.7 \times 5 \text{ m}^2 = 22 \text{ m}^2$$

Again, diameter of the hemisphere = 1.4 m

$\therefore$ Radius of the hemisphere = 0.7 m

$\therefore$ Surface area of a hemisphere

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times 0.7 \times 0.7 \text{ m}^2 = 3.08 \text{ m}^2$$

Surface area of the other hemisphere = 3.08 m^2

$\therefore$ Total surface area = $(22 + 3.08 + 3.08)\text{m}^2$

= 28.16 m^2

Rate of painting = Rs 10 per square metre

$\therefore$ Cost of painting = Rs (10 × 28.16) = Rs 281.60

PRACTICE EXERCISE 13.4A

Choose the correct option (Q 1 – 5) :

1 Mark Questions

1. The radius of a hemisphere is 2*r*. Its total surface area is :

(*a*) $4\pi r^2$ (*b*) $6\pi r^2$

(*c*) $12\pi r^2$ (*d*) $15\pi r^2$

2. The curved surface area of a hemisphere is 77cm^2. Radius of the hemisphere is :

(*a*) 3.5 cm (*b*) 7 cm

(*c*) 10.5 cm (*d*) 11 cm

3. Radius of a spehre and edge of a cube are equal. Ratio of their surface areas is : **[Imp.]**

(*a*) $\pi : 3$ (*b*) $\pi : 6$

(*c*) $2\pi : 3$ (*d*) $3 : 2\pi$

4. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is :

(*a*) 1 : 4 (*b*) 1 : 3

(*c*) 2 : 3 (*d*) 2 : 1

5. Total surface area of a hemisphere is 462 cm^2. Its curved surface area is :

(*a*) 231 cm^2 (*b*) 300 cm^2

(*c*) 308 πcm^2 (*d*) 308 cm^2

2 Marks Questions

6. The surface area of a sphere is 154 cm^2. What is its radius ?

7. The largest sphere is carved out of a cube of edge 7 cm. Find the surface area of the sphere. **[Imp.]**

3 Marks Questions

8. Find the curved surface area and the total surface area of a hemisphere of diameter 10.5 cm.

9. A spehre of radius *r* has been cut into two hemispheres. Find the ratio of the surface area of the original sphere to the total surface area of the two hemispheres.

10. The radius of a sphere is doubled. Find the increase per cent in its surface area. **[Imp.]**

13.5 VOLUME OF A CUBOID

1. The volume of an object is the measure of the space it occupies.
2. The capacity of an object is the volume of substance its interior can accommodate.
3. Volume of a cuboid of dimensions, $l \times b \times h$ is *lbh* cubic units.
4. Volume of a cube of edge *a* is a^3 cubic units.

TEXTBOOK'S EXERCISE 13.5

Q.1. *A matchbox measures* 4 *cm* × 2.5 *cm* × 1.5 *cm. What will be the volume of a packet containing* 12 *such boxes.*

Sol. Here, l = 4 cm, b = 2.5 cm, h = 1.5 cm

Volume of 1 matchbox

$= lbh = 4 \times 2.5 \times 1.5 \text{ cm}^3 = 15 \text{ cm}^3$

Volume of 12 matchboxes

$= 15 \times 12 \text{ cm}^3 = 180 \text{ cm}^3$

Q.2. *A cuboidal water tank is* 6 *m long,* 5 *m wide and* 4.5 *m deep. How many litres of water can it hold?* (1 m^3 = 1000 l)

Sol. Here, l = 6 m, b = 5 m, h = 4.5 m

Volume of the tank = lbh

$= 6 \times 5 \times 4.5 \text{ m}^3 = 135 \text{ m}^3$

= 135 × 1000 litres = 1,35,000 litres.

($\because$ 1 m^3 = 1000 litres)

Hence, the tank can hold 1,35,000 litres of water.

Q.3. *A cuboidal vessel is* 10 *m long and* 8 *m wide. How high must it be made to hold* 380 *cubic metres of a liquid?* **[2011 (T-II)]**

Sol. Here, l = 10 m, b = 8 m, h = ?

Volume of the vessel = lbh

$\Rightarrow$ 380 = 10 × 8 × h

$\Rightarrow h = \dfrac{380}{10 \times 8} = 4.75$

Hence, the tank must be made 4.75 m high

Q.4. *Find the cost of digging a cuboidal pit* 8 *m long,* 6 *m broad and* 3 *m deep at the rate of Rs* 30 *per* m^3.

Sol. l = 8 m, b = 6 m, h = 3 m

Volume of the pit

$= lbh = 8 \times 6 \times 3 \text{ m}^3 = 144 \text{ m}^3$

Cost of digging 1m^3 = Rs 30

$\therefore$ Cost of digging 144 m^3 = Rs 30 × 144

= Rs 4320

Q.5. *The capacity of a cuboidal tank is* 50000 *litres of water. Find the breadth of the tank, if its length and depth are respectively* 2.5 *m and* 10 *m.* **[Imp.]**

Sol. Here, l = 2.5 m, h = 10 m, b = ?

Capacity of the tank

$= 50000 \text{ litres} = \dfrac{50000}{1000} \text{ m}^3 = 50 \text{ m}^3$

$(\because 1 \text{ litre} = \dfrac{1}{1000} \text{ m}^3)$

Also, capacity of the tank = lbh

$\Rightarrow \quad 50 = 2.5 \times b \times 10 \Rightarrow b = \dfrac{50}{25} = 2$

Hence, breadth of the tank = 2 m

Q.6. *A village, having a population of* 4000, *requires* 150 *litres of water per head per day. It has a tank measuring* 20 *m* × 15 *m* × 6 *m. For how many days will the water of this tank last?* **[2010, 2011 (T-II)]**

Sol. Here, l = 20 m, b = 15 m, h = 6 m

Population of the village = 4000

Water consumed by 1 person in 1 day

= 150 litres

$\therefore$ Water consumed by 4000 persons in 1 day

= 4000 × 150 litres

$= \dfrac{4000 \times 150}{1000} \text{ m}^3 = 600 \text{ m}^3$

Also, capacity of the tank

$= lbh = 20 \times 15 \times 6 \text{ m}^3$

$\therefore$ Required number of days

$= \dfrac{\text{Volume of the tank}}{\text{Water consumed in 1 day}}$

$= \dfrac{20 \times 15 \times 6}{600} = 3$

Hence, the water of this tank will last for 3 days.

Q.7. *A godown measures* 40 *m* × 25 *m* × 10 *m. Find the maximum number of wooden crates each measuring* 1.5 *m* × 1.25 *m* × 0.5 *m that can be stored in the godown.*

Sol. Volume of the godown = $40 \times 25 \times 10 \text{ m}^3$

Volume 1 wooden crate = $1.5 \times 1.25 \times 0.5 \text{ m}^3$

$\therefore$ Required number of crates

$= \dfrac{\text{Volume of the godown}}{\text{Volume of 1 crate}}$

$$= \frac{40 \times 25 \times 10}{1.5 \times 1.25 \times 0.5} = 10666.67$$

Hence, the maximum number of wooden crates that can be stored in the godown = 10666

Q.8. *A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.* **[2011 (T-II)]**

Sol. Here, $a = 12$ cm

Volume of the cube = $a^3 = (12)^3$ cm^3

= 1728 cm^3

Now, volume of 1 smaller cube

$$= \frac{1728}{8} \text{ cm}^3 = 216 \text{ cm}^3$$

Let side of the new cube be A.

Then $A^3 = 216$

$\Rightarrow \quad A = \sqrt[3]{216} = 6$

Hence, side of the new cube = 6 cm

Total surface area of the bigger cube = $6\,a^2$

$= 6 \times (12)^2$ cm^2 $= 6 \times 12 \times 12$ cm^2

Total surface area of 1 smaller cube

$= 6A^2 = 6 \times 6^2$ cm^2 $= 6 \times 6 \times 6$ cm^2

Hence, required ratio = $\frac{6 \times 12 \times 12}{6 \times 6 \times 6} = \frac{4}{1} = 4 : 1$

Q.9. *A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?*

[2011 (T-II)]

Sol. Here, $b = 40$ m, $h = 3$ m, $l = 2$ km = 2000 m

Volume of water flowing through the river in 1 hour = $lbh = 2000 \times 40 \times 3$ m^3

$\therefore$ Volume of water flowing through the river in 1 minute = $\frac{2000 \times 40 \times 3}{60}$ m^3 = 4000 m^3

OTHER IMPORTANT QUESTIONS

Q.1. *How many bricks, each measuring 50 cm × 25 cm × 12 cm, will be needed to build a wall measuring 16 m × 12 m × 45 cm ?*

(*a*) 4860 (*b*) 5760

(*c*) 6320 (*d*) 8120

Sol. (*b*) Required number of bricks

$$= \frac{16 \times 100 \times 12 \times 100 \times 45}{50 \times 25 \times 12} = 5760$$

Q.2. *If the volume of a cube is* $3\sqrt{3}\,a^3$, *then surface area of the cube is :*

(*a*) $6a^2$ (*b*) $\sqrt{3}\,a^2$

(*c*) $18a^2$ (*d*) $18\sqrt{3}\,a^2$

Sol. (*c*) We have, volume

$= 3\sqrt{3}\,a^3 = \sqrt{27}\,a^3 = (\sqrt{3})^3\,a^3 = (a\sqrt{3})^3$

$\Rightarrow$ Edge of the cube = $a\sqrt{3}$

$\therefore$ Curved surface area of the cube

$= 6 \times (a\sqrt{3})^2 = 18a^2$

Q.3. *If the volume of a cube is 512 cm*^{3}, *then length of its edge is :*

(*a*) 6 cm (*b*) 8 cm

(*c*) 16 cm (*d*) 8 m **[2010]**

Sol. (*b*) We have, $a^3 = 512$

$\Rightarrow a = \sqrt[3]{512} = 8$ cm

Hence, edge of the cube is 8 cm.

Q.4. *The area of three adjacent faces of a cuboid are x, y and z sq units. If its volume is v cubic units, then :*

(*a*) $v^2 = xyz$ (b) $v = x^2y^2z^2$

(*c*) $v = \frac{xy}{z}$ (*d*) none of these

Sol. (*a*) We have, x = length × breadth, y = breadth × height and z = height × length

$\therefore\ x \times y \times z$ = (length × breadth × height)^2

= (volume)^2

Q.5. *The diagonal of a cube is* $16\sqrt{3}$ *cm. Find its volume.*

Sol. We have $a\sqrt{3} = 16\sqrt{3} \Rightarrow a = 16$ cm

$\therefore$ volume of the cube

$= a^3 = 16^3$ cm^3 = 4096 cm^3.

Q.6. *The side of a cube is 8 cm. If it is cut into smaller cubes of side 2 cm, then find the number of such cubes.* **[2010]**

Sol. Volume of the larger cube = 8^3 cm^3.

Volume of 1 smaller cube = 2^3 cm^3

Let there be n smaller cubes.

Then $n \times 2^3 = 8^3 \Rightarrow n = \frac{8^3}{2^3} = 64$

Q.7. *Find the number of cubes of sides 3 cm that can be cut out of a cuboid of dimensions 18 cm × 12 cm × 9 cm.* **[2010, 2011 (T-II)]**

Sol. Volume of the cuboid = $18 \times 12 \times 9$ cm^3

Volume of 1 cube = 3^3 cm^3

Let there be n cubes.

Then, $n \times 3^3 = 18 \times 12 \times 9$

$\Rightarrow n = \frac{18 \times 12 \times 9}{3 \times 3 \times 3} = 72$

Q. 8. *A storage tank is in the form of a cube. When it is full of water, the volume of the water is 15.625 m^3. If the present depth of water is 1.3 m, find the volume of water already used from the tank.* **[2010, 2011 (T-II)]**

Sol. Edge of the cube = $\sqrt[3]{15.625}$ m = 2.5 m

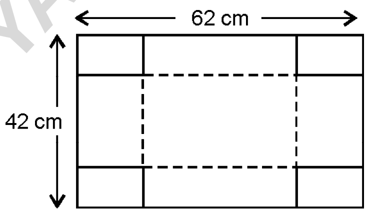
Volume of water present in the tank

= $2.5 \times 2.5 \times 1.3$ m^3 = 8.125 m^3

$\therefore$ Volume of water used

= (15.625 – 8.125) m^3 = 7.5 m^3

Q.9. *A rectangular metallic sheet is 62 cm long and 42 cm wide. From each of its corners, a square of side 6 cm is cut out as shown. An open box is made by folding along the dotted lines from the remaining sheet. Find the volume of the rectangular box.* **[2010]**



Sol. Clearly, height of the box = 6 cm

Length of the box

= {(62 – (6 + 6)} cm = 50 cm

Width of the box

= {(42 – (6 + 6)} cm = 30 cm

$\therefore$ Volume of the box = length × width × height = $50 \times 30 \times 6$ cm^3 = 9000 cm^3

Q.10. *Three metal cubes whose edges measures 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. Find its edge. Also, find the surface area of new cube formed.* **[2010]**

Sol. Volumes of the cubes with edges 3 cm, 4 cm and 5 cm respectively are 3^3 cm^3, 4^3 cm^3 and 5^3 cm^3.

Total volume of the three cubes

= $(3^3 + 4^3 + 5^3)$ cm^3 = 216 cm^3

$\therefore$ Volume of the new cube = 216 cm^3

$\Rightarrow a^3 = 216 \Rightarrow a = \sqrt[3]{216} = 6$ cm

Surface area of the new cube = $6a^2$

= 6×6^2 cm^2 = 216 cm^2

Q.11. *The weight of a metallic cuboid is 112 kg. Its width is 28 cm and length is 32 cm. If 1 cm^3 of the metal weighs 25 g, find the height of the cuboid.* **[2010]**

Sol. Volume of the cuboid = $\frac{112}{0.025}$ cm^3

= 4480 cm^3

$\therefore l \times b \times h = 4480$

$\Rightarrow 32 \times 28 \times h = 4480$

$\Rightarrow h = \frac{4480}{32 \times 28}$ cm = 5 cm

Q.12. *A solid cube of side 12 cm is cut into 8 cubes of equal volumes. Find the side of new cube.* **[2010]**

Sol. Volume of the bigger cube = 12^3 cm^3

Let the edge of each small cube be a.

Then, volume of 8 small cubes = $8 \times a^3$

$\therefore 8a^3 = 12^3$

$\Rightarrow a^3 = \frac{12 \times 12 \times 12}{8} = 216 \Rightarrow a = 6$ cm

Q.13. *Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted down into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.* **[2010, 2011 (T-II)]**

Sol. Diagonal of the cube formed

= $12\sqrt{3}$ cm^3

$\therefore$ Edge of the the cube formed = 12 cm

Now, volume of the the cube formed = 12^3 cm^3

$\therefore$ $(3x)^3 + (4x)^3 + (5x)^3 = 12 \times 12 \times 12$

$\Rightarrow 27x^3 + 64x^3 + 125x^3 = 12 \times 12 \times 12$

$\Rightarrow x^3 = \dfrac{12 \times 12 \times 12}{216} = 8 \Rightarrow x = 2.$

Hence, edges of the cubes are 3 × 2 cm, 4 × 2 cm and 5 × 2 cm or 6 cm, 8 cm and 10 cm.

Q.14. *Water in a rectangular reservoir having base* 80 *m* × 60 *m is* 6.5 *m deep. In what time can the water be emptied by a pipe of which the cross section is a square of side* 20 *cm, if the water runs through the pipe at the rate of* 15 *km/h?*

Sol. Area of the cross section of the pipe

= 400 cm^2 = 0.04 m^2

Rate of the flow of water through the pipe

= 15 km/h = 15 × 1000 m/h

Volume of water emptied by the pipe in 1 hour

= 0.04 × 15 × 1000 m^3

$\therefore$ Time taken by the pipe to empty the tank

$= \dfrac{80 \times 60 \times 6.5}{0.04 \times 15 \times 1000}$ hours = 52 hours

PRACTICE EXERCISE 13.5A

Choose the correct option (Q 1 – 5) :

1 Mark Questions

1. The lateral surface area of a cube is 256 m^2. The volume of the cube is :

(*a*) 512 m^3 (*b*) 64 m^3

(*c*) 216 m^3 (*d*) 256 m^3

2. The number of planks of dimensions (4 m × 5 m × 2 m) that can be stored in a pit which is 40 m long, 12 m wide and 160 m deep is :

(*a*) 1900 (*b*) 1920

(*c*) 1800 (*d*) 1840

3. The total surface area of a cube is 96 cm^2. The volume of the cube is :

(*a*) 8 cm^3 (*b*) 512 cm^3

(*c*) 64 cm^3 (*d*) 27 cm^3

4. The dimensions of a cuboid are in the ratio 3 : 2 : 1. If its volume is 1296 cm^3, the dimensions of the cuboid are : **[Imp.]**

(*a*) 6 cm, 4 cm, 2 cm

(*b*) 12 cm, 8 cm, 4 cm

(*c*) 15 cm, 10 cm, 5 cm

(*d*) 18 cm, 12 cm, 6 cm

5. The lateral surface area of a cube is 400 cm^2 . The volume of the cube is :

(*a*) 100 cm^3 (*b*) 1000 cm^3

(*c*) 500 cm^3 (*d*) 600 cm^3

2 Marks Questions

6. The length of the diagonal of a cube is $15\sqrt{3}$ cm. Find its volume.

7. If the edge of a cube is doubled, what is the ratio of the volume of the first cube to that of the second cube? **[2011 (T-II)]**

3 Marks Questions

8. Find the depth of a tank , which has a rectangular base measuring 6 m × 4 m and holds as much water as another tank whose dimensions are 8 m × 5 m × 3 m.

9. If the area of three adjacent faces of a rectangular block are in the ratio. 2 : 3 : 4. and its volume is 9000 cm^3, find the dimensions of the block. **[HOTS]**

10. The outer dimensions of a closed wooden box are 10 cm × 8 cm × 7 cm. The thickness of the wood is 1 cm. Find the total cost of wood required to make the box, if 1 cm^3 of wood costs Rs 2. **[2011 (T-II)]**

13.6 VOLUME OF A CYLINDER

1. Volume of a right circular cylinder of base radius r and height h = area of base × height = $\pi r^2 h$.

TEXTBOOK'S EXERCISE 13.6

Q.1. *The circumference of the base of a cylindrical vessel is* 132 *cm and its height is* 25 *cm. How many litres of water can it hold?* (1000 cm^3 = 1l) **[2011 (T-II)]**

Sol. Here, h = 25 cm, $2\pi r$ = 132 cm.

$$2\pi r = 132 \Rightarrow 2 \times \frac{22}{7} \times r = 132$$

$$\Rightarrow r = \frac{132 \times 7}{2 \times 22} \text{ cm} = 21 \text{ cm}$$

Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3 = 34650 \text{ cm}^3$$

$$= \frac{34650}{1000} \text{ litres} = 34.65 \text{ litres}$$

Q.2. *The inner diameter of a cylindrical wooden pipe is* 24 *cm and its outer diameter is* 28 *cm. The length of the pipe is* 35 *cm. Find the mass of the pipe, if* 1 cm^3 *of wood has a mass of* 0.6 *g.* **[2010]**

Sol. Here, inner radius (r) = $\frac{24}{2}$ cm = 12 cm

Outer radius (R) = $\frac{28}{2}$ cm = 14 cm, h = 35 cm

Volume of the wood used in the pipe

$= \pi(R^2 - r^2)\, h$

$$= \frac{22}{7} [(14)^2 - (12)^2] \times 35 \text{ cm}^3$$

$$= \frac{22}{7} \times 26 \times 2 \times 35 \text{ cm}^3 = 5720 \text{ cm}^3$$

Mass of 1 cm^3 of wood = 0.6 g

$\therefore$ Mass of 5720 cm^3 of wood

= 0.6 × 5720 g = 3432 g

= 3.432 kg

Q.3. *A soft drink is available in two packs — (i) a tin can with a rectangular base of length* 5 *cm and width* 4 *cm, having a height of* 15 *cm and (ii) a plastic cylinder with circular base of diameter* 7 *cm and height* 10 *cm. Which container has greater capacity and by how much?* **[2010]**

Sol. For tin can with rectangular base

l = 5 cm, b = 4 cm, h = 15 cm

Volume of the tin can

$= lbh = 5 \times 4 \times 15 \text{ cm}^3 = 300 \text{ cm}^3$

For plastic cylinder with circular base

$r = \frac{7}{2}$ cm = 3.5 cm, h = 10 cm

Volume of the plastic cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 10 \text{ cm}^3 = 385 \text{ cm}^3$$

Difference in the capacities of the two containers = (385 – 300) cm^3 = 85 cm^3

Hence, the plastic cylinder with circular base has greater capacity by 85 cm^3.

Q.4. *If the lateral surface of a cylinder is* 94.2 cm^2 *and its height is* 5 *cm, then find (i) radius of its base (ii) its volume*

(Use π = 3.14)

Sol. Here, h = 5 cm, $2\pi rh$ = 94.2 cm^2.

(i) $2\pi rh = 94.2$

$\Rightarrow 2 \times 3.14 \times r \times 5 = 94.2$

$$\Rightarrow r = \frac{94.2}{2 \times 3.14 \times 5} = 3$$

Hence, base radius of the cylinder = 3 cm

(ii) Volume of the cylinder

$= \pi r^2 h = 3.14 \times 3 \times 3 \times 5 \text{ cm}^3 = 141.3 \text{ cm}^3$

Q.5. *It costs Rs* 2200 *to paint the inner curved surface of a cylindrical vessel* 10 *m deep. If the cost of painting is at the rate of Rs* 20 *per* m^2, *find :* **[2010 (T-II)]**

(i) inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

Sol. Here, h = 10 m

(i) Inner curved surface area

$$= \frac{\text{Total cost}}{\text{Cost of painting per m}^2} = \frac{2200}{20} \text{m}^2 = 110 \text{ m}^2$$

(ii) We have, $2\pi rh = 110$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 10 = 110$$

$$\Rightarrow r = \frac{110 \times 7}{2 \times 22 \times 10} = 1.75 \text{ m}$$

(iii) Capacity of the vessel = $\pi r^2 h$

$= \frac{22}{7} \times 1.75 \times 1.75 \times 10 \text{ m}^3 = 96.25 \text{ m}^3$

$= 96.25$ kl   [$1 \text{ m}^3 = 1$ *kl*]

Q.6. *The capacity of a closed cylindrical vessel of height* 1 *m is* 15.4 *litres. How many square metres of metal sheet would be needed to make it?* **[2011 (T-II)]**

Sol. Here, $h = 1$ m, volume = 15.4 litres

$= \frac{15.4}{1000} \text{ m}^3 = 0.0154 \text{ m}^3$

Also, volume of the cylindrical vessel = $\pi r^2 h$

$\Rightarrow 0.0154 = \frac{22}{7} \times r^2 \times 1$

$\Rightarrow r^2 = \frac{0.0154 \times 7}{22} = 0.0049 \Rightarrow r = 0.07$ m

$\therefore$ Total surface area of the cylinder

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 0.07 (1 + 0.07) \text{ m}^2$

$= 44 \times 0.01 \times 1.07 \text{ m}^2 = 0.4708 \text{ m}^2$

Hence, 0.4708 m^2 of metal sheet would be needed.

Q.7. *A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is* 7 *mm and the dimeter of the graphite is* 1 *mm. If the length of the pencil is* 14 *cm, find the volume of the wood and that of the graphite.* **[2011 (T-II)]**

Sol. Here, $h = 14$ cm.

Radius of the pencil (R) = $\frac{7}{2}$ mm = 0.35 cm.

Radius of the graphite (r) = $\frac{1}{2}$ mm = 0.05 cm.

Volume of the the graphite

$= \pi r^2 h = \frac{22}{7} \times 0.05 \times 0.05 \times 14 \text{ cm}^3$

$= 0.11 \text{ cm}^3$

Volume of the the wood = $\pi(R^2 - r^2)h$

$= \frac{22}{7} \times [(0.35)^2 - (0.05)^2] \times 14 \text{ cm}^3$

$= \frac{22}{7} \times 0.4 \times 0.3 \times 14 \text{ cm}^3 = 5.28 \text{ cm}^3$

Hence, volume of the wood = 5.28 cm^3 and volume of the graphite = 0.11 cm^3

Q.8. *A patient in a hospital is given soup daily in a cylindrical bowl of diameter* 7 *cm. If the bowl is filled with soup to a height of* 4 *cm, how much soup the hospital has to prepare daily to serve* 250 *patients?*

Sol. Here, $r = \frac{7}{2}$ cm = 3.5 cm, $h = 4$ cm

Capacity of 1 cylindrical bowl = $\pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 4 \text{ cm}^3 = 154 \text{ cm}^3$

Hence, soup consumed by 250 patients per day = $250 \times 154 \text{ cm}^3 = 38500 \text{ cm}^3$

OTHER IMPORTANT QUESTIONS

Q.1. *The radius of a wire is decreased to one third. If the volume remains the same, its length will increase by :* **[Imp.]**

(*a*) 2 *times* (*b*) 3 *times*
(*c*) 6 *times* (*d*) 9 *times*

Sol. (*d*) Let the new length be x.

Then, $\pi \times r^2 \times h = \pi \times \left(\frac{r}{3}\right)^2 \times x \Rightarrow 9h = x$

Q.2. *If the radius of a wire is decreased to one-fourth of its original and its volume remains same, then how many times will the new length becomes its original length ?* **[Imp.]**

(*a*) 4 *times* (*b*) 8 *times*
(*c*) 16 *times* (*d*) 20 *times*

Sol. (*c*) Let the original radius be r. Then, volume = $\pi r^2 h$

New radius = $\frac{r}{4}$ and new length = H

Then, $\pi r^2 h = \pi\left(\frac{r}{4}\right)^2 \times H \Rightarrow H = 16h$

Q.3. *The volume of a cylinder is* 567π *cm*^{3} *and its height is* 7 *cm. Its total surface area is :*

(*a*) 280π *cm*^{2} (*b*) 288π *cm*^{2}
(*c*) 340π *cm*^{2} (*d*) 480π *cm*^{2}

Sol. (b) We have, $\pi r^2 \times 7 = 567\pi$

$$\Rightarrow r^2 = \frac{567}{7} = 81 \Rightarrow r = 9 \text{ cm}$$

$\therefore$ Total surface area

$$= 2\pi \times 9(7 + 9) \text{ cm}^2 = 288\,\pi \text{ cm}^2.$$

Q.4. *The base radii of two cylinders are in the ratio* 2 : 3 *and their heights are in the ratio* 3 : 2. *The ratio of their volumes is:* **[Imp.]**

(a) 2 : 3 (b) 3 : 2

(c) 1 : 3 (d) 2 : 1

Sol. (a) Required ratio

$$= \frac{\pi \times (2x)^2 \times 3y}{\pi \times (3x)^2 \times 2y} = \frac{12}{18} = \frac{2}{3} = 2 : 3$$

Q.5. *If the radius of a cylinder is doubled and height is halved, what will be the volume of the new cylinder?*

Sol. Volume of the original cylinder = $\pi r^2 h$

Volume of the new cylinder

$= \pi \times (2r)^2 \times \left(\frac{h}{2}\right) = 2\pi r^2 h$

= twice the volume of the original cylinder.

Q.6. *How many litres of water flow out of a pipe having an area of cross section of* 5 *cm*^{2} *in one minute, if the speed of water in the pipe is* 30 *cm/sec ?*

Sol. Volume of water which flows out of the pipe in 1 second = 5×30 cm^3

$\therefore$ Volume of water which flows out of the pipe in 1 minute = $5 \times 30 \times 60$ cm^3 = 9000 cm^3

$= \frac{9000}{1000}$ litres = 9 litres

Q.7. *Two cylindrical vessels have their base radii as* 16 *cm and* 8 *cm respectively. If their heights are* 8 *cm and* 16 *cm respectively, then find the ratio of their volumes.* **[2010]**

Sol. Volume of first vessel

$$= \pi r_1^2 h_1 = \frac{22}{7} \times 16 \times 16 \times 8 \text{ cm}^3$$

Volume of the second vessel

$$= \pi r_2^2 h_2 = \frac{22}{7} \times 8 \times 8 \times 16 \text{ cm}^3$$

$$\therefore \text{ Required ratio } = \frac{\frac{22}{7} \times 16 \times 16 \times 8}{\frac{22}{7} \times 8 \times 8 \times 16} = 2 : 1$$

Q.8. *The pillars of a temple are in the shape of a cylinder. If each pillar has base radius* 20 *cm and height* 10 *cm, find the volume of concrete required to build* 7 *such pillars.* **[2010, 2011 (T-II)]**

Sol. We have, r = 20 cm, h = 10 cm

Volume of 1 pillar

$$= \pi r^2 h = \frac{22}{7} \times 20 \times 20 \times 10 \text{ cm}^3$$

Volume of 7 pillars = $7 \times \frac{22}{7} \times 20 \times 20 \times 10$ cm^3 = 88000 cm^3

Hence, 88000 cm^3 of concrete is required to build 7 pillars.

Q.9. *The radius and height of a cylinder are in the ratio* 2 : 3. *If the volume of the cylinder is* 1617 *cm*^{3}, *find its radius and height.*

[2010, 2011 (T-II)]

Sol. Let the radius and height of the cylinder be $2x$ and $3x$ respectively.

Then, volume of the cylinder = $\pi r^2 h = \pi \times 2x \times 2x \times 3x$

$$\Rightarrow 1617 = \frac{22}{7} \times 12x^3$$

$$\Rightarrow x^3 = \frac{1617 \times 7}{22 \times 12} = \frac{343}{8} \Rightarrow x = \frac{7}{2} = 3.5$$

$\therefore$ Radius of the cylinder = $2x$ = 7 cm

And, height of the cylinder = $3x$ = 10.5 cm

Q.10. *The sum of height and radius of the base of a solid cylinder is* 37 *m. Total surface area of the cylinder is* 1628 *m*^{2}. *Find its volume.*

[2010, 2011 (T-II)]

Sol. We have, $2\pi r(h + r) = 1628$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 37 = 1628$$

[Given $h + r$ = 37 m]

$$\Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7 \text{ m}$$

Now, $h + r = 37 \Rightarrow h = (37 - 7)$ m = 30 m

$\therefore$ Volume of the cylinder

$$= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 \text{ m}^3 = 4620 \text{ m}^3$$

Q.11. *The curved surface area of a cylinder is* 5500 cm^2 *and the circumference of the base is* 110 *cm. Find the height and volume of the cylinder.* **[2010, 2011 (T-II)]**

Sol. We have, $2\pi r = 110$

$$\Rightarrow r = \frac{110 \times 7}{2 \times 22} = 17.5 \text{ m}$$

Also, curved surface area of the cylinder $= 2\pi rh$

$$\Rightarrow 5500 = 110 \times h \Rightarrow h = \frac{5500}{110} = 50$$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 17.5 \times 17.5 \times 50 \text{ cm}^3 = 48125 \text{ cm}^3$$

Q.12. *The radius of the base of a right circular cylinder is* 20 *cm and its volume is* 17600 cm^3. *Find the total surface area of the cylinder.* **[2011 (T-II)]**

Sol. We have $\pi r^2 h = 17600$

$$\Rightarrow \frac{22}{7} \times 20 \times 20 \times h = 17600$$

$$\Rightarrow h = \frac{17600 \times 7}{22 \times 20 \times 20} = 14$$

Total surface area of the cylinder $= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 20\,(14 + 20) \text{ cm}^3$$

$$= \frac{40 \times 22 \times 34}{7} \text{ cm}^3 = 4274\frac{2}{7} \text{ cm}^3$$

Q.14. *Outer and inner diameter of a cylindrical wooden pipe are* 28 *cm and* 24 *cm respectively. Find the mass of pipe of length if pipe is* 35 *cm. Find the volume of wood used.* **[2010, 2011 (T-II)]**

Sol. Outer radius (R) of the pipe = 14 cm
Inner radius (r) of the pipe = 12 cm.
Height (h) of the pipe = 35 cm
Volume of wood used to make the pipe
$= \pi (R^2 - r^2)\, h$

$$= \frac{22}{7} \times (14^2 - 12^2) \times 35 \text{ cm}^3$$

$$= \frac{22}{7} \times (14 + 12)(14 - 12) \times 35 \text{ cm}^3$$

$$= 22 \times 26 \times 2 \times 5 \text{ cm}^3 = 5720 \text{ cm}^3$$

Q.15. *The radius and height of a cylinder are in the ratio* 5 : 7 *and its volume is* 550 cm^3. *Find its diameter. (Use* $\pi = \frac{22}{7}$) **[2011 (T-II)]**

Sol. Let the radius of the base and height of the cylinder be $5x$ cm and $7x$ cm respectively. Then, volume = 550 cm^3

$$\Rightarrow \frac{22}{7} \times (5x)^2 \times 7x = 550$$

[Use $r = 5x$, $h = 7x$ and volume $\pi r^2 h$]

$$\Rightarrow \frac{22}{7} \times 25x^2 \times 7x = 550 \Rightarrow 22 \times 25x^3 = 550$$

$\Rightarrow x^3 = 1 \Rightarrow x = 1$
$\therefore$ Radius $= 5x$ cm $= 5 \times 1$ cm $= 5$ cm
Hence, diameter $= 2 \times 5$ cm $= 10$ cm

Q.16. *A solid cylinder has a total surface area of* 462 m^2. *Its curved surface area is one third of its total surface area. Find the volume of the area of cylinder.* [*Take* $\pi = \frac{22}{7}$]**[2011 (T-II)]**

Sol. Let r be the radius of the base and h be the height of the cylinder. Then,

Total surface area $= 2\pi r(h + r)$ cm^2, curved surface area $= 2\pi rh$ cm^2.

We have, $2\pi rh = \frac{1}{3} \times 462$ cm^2

$\Rightarrow 2\pi rh = 154$ cm^2 ... (i)
Again, $2\pi rh + 2\pi r^2 = 462$
$\Rightarrow 154 + 2\pi r^2 = 462$
$\Rightarrow 2\pi r^2 = 462 - 154 \Rightarrow r^2 = 49 \Rightarrow r = 7$ cm

From (i), we have, $h = \frac{7}{2}$ cm

Hence, volume $= \pi r^2 h$

$$= \frac{22}{7} \times 7^2 \times \frac{7}{2} \text{ cm}^3 = 539 \text{ cm}^3$$

PRACTICE EXERCISE 13.6A

Choose the correct option (Q 1 – 6) :

1 Mark Questions

1. In a cylinder, if radius is halved and height is doubled, the volume will be :

(*a*) same (*b*) doubled

(*c*) halved (*d*) four times

2. The volume of a cylinder of radius x and height $2x$ is :

(*a*) $\frac{\pi x^3}{2}$ (*b*) $2\pi x^2$

(*c*) $2\pi x^3$ (*d*) πx^3

3. Curved surface area of a cylinder of base radius $2r$ is πr^2. Volume of the cylinder is :

(*a*) $3\pi r^3$ (*b*) $\frac{\pi r^2}{2}$

(*c*) πr^3 (*d*) $\frac{\pi r^3}{4}$

4. The circumference of the base of a cylinder is 44 cm and its height is 8 cm. The volume of the cylinder is :

(*a*) 1322 cm^3 (*b*) 1200 cm^3

(*c*) 1232 cm^3 (*d*) 1321 cm^3

5. The thickness of a hollow cylinder is 1 cm. It is 7 cm long and its inner radius is 3 cm. The volume of the wood required to make the cylinder is :

(*a*) 154π cm^3 (*b*) 514 cm^3

(*c*) 154 cm^3 (*d*) 145π cm^3

6. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is : **[Imp.]**

(*a*) 10 : 17 (*b*) 20 : 27

(*c*) 17 : 27 (*d*) 20 : 37

2 Marks Questions

7. Check whether the following statement is true or not.

If the radius of a right circular cylinder is halved and height is doubled, the volume will remain unchanged.

8. A rectangular sheet of paper 44 cm × 20 cm, is rolled along its length to form a cylinder. Find the volume of the cylinder, thus formed.

3 Marks Questions

9. The curved surface area of a cylinder is 4400 cm^2 and the circumference of the base is 110 cm. Find the volume of the cylinder. **[2010]**

10. Water flows through a pipe of radius 0.6 cm at the rate of 8 cm/sec. This pipe is draining out water from a tank which holds 1000 litres of water when full. How long would it take to completely empty the tank? **[HOTS]**

11. How many coins 1.75 cm in diameter and 2 mm thick must be melted down to form a rectangular solid whose dimensions are 11 cm × 10 cm × 7 cm?

4 Marks Questions

12. Rain water which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel, if the rainfall is 1 cm?

13. A copper wire of diameter 6 mm is evenly wraped on a cylinder of length 18 cm and diameter 49 cm to cover its whole surface. Find the length of the wire. **[HOTS]**

14. The difference between the outside and inside surfaces of a cylindrical pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cm^3 of metal, find the outer and inner radii of the pipe. **[HOTS]**

13.7 VOLUME OF A RIGHT CIRCULAR CONE

1. Volume of a right circular cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$.

TEXTBOOK'S EXERCISE 13.7

Q.1. *Find the volume of the right circular cone with*

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm

Sol. (i) Here, $r = 6$ cm, $h = 7$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7 \text{ cm}^3 = 264 \text{ cm}^3$

(ii) Here, $r = 3.5$ cm, $h = 12$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12 \text{ cm}^3 = 154 \text{ cm}^3$

Q.2. *Find the capacity in litres of a conical vessel with*

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 13 cm

Sol. (i) Here, $r = 7$ cm, $l = 25$ cm

$\therefore h = \sqrt{l^2 - r^2}$

$= \sqrt{625 - 49} \text{ cm} = \sqrt{576} \text{ cm} = 24 \text{ cm}$

Volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ cm}^3 = 1232 \text{ cm}^3$

$= \frac{1232}{1000}$ litres $= 1.232$ litres

(ii) Here, $h = 12$ cm, $l = 13$ cm

$\therefore r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2}$ cm

$= \sqrt{169 - 144} \text{ cm} = \sqrt{25} \text{ cm} = 5 \text{ cm}$

Volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 \text{ cm}^3$

$= \frac{22 \times 5 \times 5 \times 4}{7} \text{ cm}^3$

$= \frac{22 \times 5 \times 5 \times 4}{7 \times 1000}$ litres $= \frac{11}{35}$ litres

Q.3. *The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$)* **[2011 (T-II)]**

Sol. (i) Here, $h = 15$ cm, volume $= 1570 \text{ cm}^3$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow 1570 = \frac{1}{3} \times 3.14 \times r^2 \times 15$

$\Rightarrow r^2 = \frac{1570 \times 3}{3.14 \times 15} = 100 \Rightarrow r = 10$

Hence, radius of the base = 10 cm

Q.4. *If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base.* **[2011 (T-II)]**

Sol. Here, $h = 9$ cm, volume $= 48\pi \text{ cm}^3$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow 48\pi = \frac{1}{3}\pi \times r^2 \times 9$

$\Rightarrow r^2 = \frac{48\,\pi \times 3}{\pi \times 9} = 16 \Rightarrow r = 4$

Hence, base diameter of the cone

$= 2 \times 4 \text{ cm} = 8 \text{ cm}$

Q.5. *A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?*

Sol. Here, $r = \frac{3.5}{2}$ m $= 1.75$ m, $h = 12$ m

Capacity of the pit $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 12 \text{ m}^3$

$= 38.5 \text{ m}^3 = 38.5$ *kl*

Q.6. *The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find*

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone.

Sol. Here, $r = \frac{28}{2}$ cm = 14 cm,
volume = 9856 cm^3

(i) Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$\Rightarrow 9856 = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h$$

$$\Rightarrow h = \frac{9856 \times 3 \times 7}{22 \times 14 \times 14} = 48$$

Hence, height of the cone = 48 cm

(ii) Slant height $l = \sqrt{h^2 + r^2}$

$$= \sqrt{(48)^2 + (14)^2} \text{ cm} = \sqrt{2304 + 196} \text{ cm}$$

$$= \sqrt{2500} \text{ cm} = 50 \text{ cm}$$

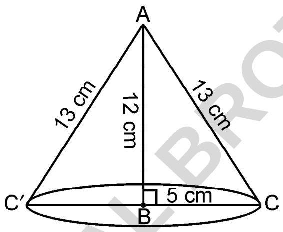
Hence, slant height of the cone = 50 cm

(iii) Curved surface area of the cone

$$= \pi r l = \frac{22}{7} \times 14 \times 50 \text{ cm}^2 = 2200 \text{ cm}^2$$

Q.7. *A right triangle ABC with sides* 5 *cm,* 12 *cm and* 13 *cm is revolved about the side* 12 *cm. Find the volume of the solid so obtained.*

Sol. The solid formed is a cone, whose height h = 12 cm, base radius r = 5 cm.



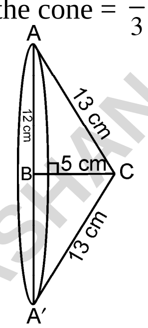
$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 12 \text{ cm}^3 = 100\,\pi \text{ cm}^3$$

Q.8. *If the triangle ABC in the Question* 7 *above is revolved about the side* 5 *cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in questions* 7 *and* 8. **[2011 (T-II)]**

Sol. Here, radius r of the cone = 12 cm and height h of the cone = 5 cm

$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h$



$$= \frac{1}{3}\pi \times 12 \times 12 \times 5 = 240\,\pi\text{cm}^3$$

Hence, required ratio $= \frac{100\,\pi}{240\,\pi} = \frac{5}{12} = 5 : 12$

Q.9. *A heap of wheat is in the form of a cone whose diameter is* 10.5 *m and height is* 3 *m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.*

Sol. Here, radius $r = \frac{10.5}{2}$ m = 5.25 m, h = 3 m

Volume of the heap = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3 \text{ m}^3 = 86.625 \text{ m}^3$$

Now, $l = \sqrt{h^2 + r^2} = \sqrt{3^2 + (5.25)^2}$

$$= \sqrt{9 + 27.5625} = \sqrt{36.5625} = 6.05 \text{ m (approx)}$$

Curved surface area of the cone = $\pi r l$

$$= \frac{22}{7} \times 5.25 \times 6.05 \text{ m}^2 = 99.825 \text{ m}^2$$

Hence, 99.825 m^2 of canvas is needed.

OTHER IMPORTANT QUESTIONS

Q.1. *If the slant height of a cone is double its base radius, then volume of the cone is :*

(*a*) $\frac{1}{\sqrt{3}}\pi$ *(radius)*^{3} (*b*) $\sqrt{3}\,\pi$ *(radius)*^{3}

(*c*) π *(radius)*^{3} (*d*) $\sqrt{2}\,\pi$ *(radius)*^{3}

Sol. (*a*) Height of the cone

$$= \sqrt{(2x)^2 - x^2} = x\sqrt{3}$$

$\therefore$ Required volume of the cone

$$= \frac{1}{3}\pi \times x^2 \times x\sqrt{3} = \frac{\pi x^3}{\sqrt{3}} = \frac{1}{\sqrt{3}}\pi \text{ (radius)}^3$$

Q.2. *Ratio of the volume of a cone and a cylinder of same radius of base and same height is :* **[2010]**

(*a*) 1 : 1 (*b*) 1 : 2
(*c*) 1 : 3 (*d*) 1 : 4

Sol. (*c*) Required ratio

$$= \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 h} = \frac{1}{3} = 1 : 3$$

Q.3. *Two cones have the same radius but their heights are in the ratio* 1 : 3*. The ratio of their volumes is :* **[2010]**

(*a*) 1 : 3 (*b*) 1 : 9
(*c*) 9 : 1 (*d*) 3 : 1

Sol. (*a*) Required ratio $= \dfrac{\frac{1}{3}\pi r^2 \times h}{\frac{1}{3}\pi r^2 \times 3h} = \dfrac{1}{3} = 1 : 3$

Q.4. *If the radius of a cone is increased by* 100% *and height is decreased by* 50%*, then the volume of the new cone is .... the volume of the original cone.*

(*a*) *twice* (*b*) *thrice*
(*c*) *half* (*d*) *unchanged*

Sol. (*a*) Original volume of the cone $= \frac{1}{3}\pi r^2 h$

Volume of the new cone $= \frac{1}{3}\pi \times (2r)^2 \times \left(\frac{h}{2}\right)$

$= \frac{1}{3}\pi \times 2r^2 h = 2\left(\frac{1}{3}\pi r^2 h\right)$

= 2 × (original volume).

Q.5. *An edge of a cube measures r cm. If the largest possible right circular cone is cut out of this cube, then find the volume of the cone.* **[Imp.]**

Sol. Radius of the cone $= \frac{r}{2}$ cm and height of the cone = r cm

∴ Volume of the cone

$$= \frac{1}{3}\pi \times \frac{r^2}{4} \times r \text{ cm}^3 = \frac{\pi r^3}{12} \text{ cm}^3.$$

Q.6. *Find the volume of a right circular cone with radius* 6 *cm and height* 14 *cm* $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[2010]**

Sol. We have, r = 6 cm, h = 14 cm

∴ Volume of the cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 14 \text{ cm}^3 = 528 \text{ cm}^3$

Q.7. *If the area of the base of a cone is* 420 *cm*^{2} *and its height is* 15 *cm, then find its volume* **[2010]**

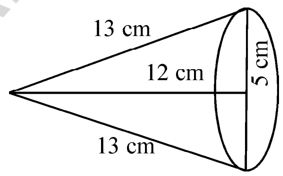
Sol. We have, πr^2 = 420 cm^2, h = 15 cm.

∴ Volume of the cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 420 \times 15 \text{ cm}^3 = 2100 \text{ cm}^3$

Q.8. *A right triangle with its sides* 13 *cm,* 12 *cm and* 5 *cm is rotated about the side* 12 *cm. Find the volume of the solid so generated.* **[2010]**

Sol. Radius of the cone = 5 cm and height of the cone = 12 cm



∴ Volume of the cone $= \frac{1}{3}\pi \times 5^2 \times 12 \text{ cm}^3$

$= 100 \times \frac{22}{7} \text{ cm}^3 = 314.28 \text{ cm}^3$

Q.9. *Radius and height of a cone are in the ratio* 4 : 5*. The area of base is* 154 *cm*^{2}*. Find the volume of the cone.* **[2010, 2011 (T-II)]**

Sol. Let the radius and height of the cone be $4x$ and $5x$ respectively. Then, area of base = 154 cm^2

$\Rightarrow \pi \times (4x)^2 = 154 \Rightarrow x^2 = \frac{154 \times 7}{22 \times 16} = \frac{49}{16}$

$\Rightarrow x = \frac{7}{4} = 1.75$

∴ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (4x)^2 \times 5x$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times \frac{49}{16} \times 5 \times 1.75 \text{ cm}^3$

$= \frac{385}{6} \text{ cm}^3.$

Q.10. *From a right circular cylinder with height* 15 *cm and radius* 7 *cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid.* **[2010]**

Sol. Volume of the cylinder = $\pi r^2 h$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\therefore$ Volume of the remaining solid

$= (\pi r^2 h - \frac{1}{3}\pi r^2 h) = \frac{2}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 15 \text{ cm}^3 = 1540 \text{ cm}^3$

Q.11. *Volume of a right circular cone is* 9856 *cm*^{3}. *If diameter of the cone is* 28 *cm, find the curved surface area of the cone.* **[2010]**

Sol. We have, $r = \frac{28}{2}$ cm = 14 cm

Now, $\frac{1}{3}\pi r^2 h = 9856$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$

$\Rightarrow h = \frac{9856 \times 3 \times 7}{196 \times 22} = 48$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{196 + 2304}$ cm $= \sqrt{2500}$ cm = 50 cm

$\therefore$ Curved surface area of the cone = πrl

$= \frac{22}{7} \times 14 \times 50 \text{ cm}^2 = 2200 \text{ cm}^2$

Q.12. *If h, c, v are respectively the height, the curved surface and the volume of a cone, show that* $3\pi vh^3 - c^2h^2 + 9v^2 = 0$ **[V. Imp.]**

Sol. Slant height of the cone = $\sqrt{r^2 + h^2}$, where r is the base radius.

Now, $c = \pi r\sqrt{r^2 + h^2} \Rightarrow c^2 = \pi^2 r^2 (r^2 + h^2)$ ... (i)

And $v = \frac{1}{3}\pi r^2 h \Rightarrow r^2 = \frac{3v}{\pi h}$ ... (ii)

Substituting the values of r^2 from (ii) into (i), we get

$$c^2 = \pi^2 \frac{3v}{\pi h}\left(\frac{3v}{\pi h} + h^2\right) = \frac{3v\pi}{h}\left(\frac{3v + \pi h^3}{\pi h}\right)$$

$\Rightarrow c^2h^2 = 3v(3v + \pi h^3) = 9v^2 + 3\pi vh^3$

$\Rightarrow 3\pi vh^3 - c^2h^2 + 9v^2 = 0$

Q.13. *A semicircular sheet of diameter* 28 *cm is bent to make a conical shape. Find the volume of the cone so formed.* **[2010, 2011 (T-II)]**

Sol. Slant height of the cone

= radius of the semi-circular sheet = 14 cm

Circumference of the base of the cone

$= \pi \times 14 \text{ cm} = \frac{22}{7} \times 14 \text{ cm} = 44 \text{ cm}$

$\Rightarrow 2\pi r = 44 \Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7$ cm

Also, height of the cone

$h = \sqrt{l^2 - r^2} = \sqrt{14^2 - 7^2}$ cm

$= \sqrt{147}$ cm $= 7\sqrt{3}$ cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\sqrt{3} \text{ cm}^3$

$= \frac{1078\sqrt{3}}{3} \text{ cm}^3 = 622.38 \text{ cm}^3$

Q.14. *The radius of a cone is* 5 *cm and height is* 12 *cm. Find the curved surface area and volume of the cone.* **[2010]**

Sol. We have, $r = 5$ cm, $h = 12$ cm,

$\therefore l = \sqrt{h^2 + r^2} = \sqrt{144 + 25}$ cm = 13 cm

Curved surface area of the cone

$= \pi rl = \frac{22}{7} \times 5 \times 13 \text{ cm}^2 = 204.28 \text{ cm}^2$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 \text{ cm}^3 = 314.28 \text{ cm}^3$

Q.15. *A heap of wheat is in the form of a cone. Diameter of the base of the heap is* 8 *m and height is* 3 *m. Find its volume. The heap is to be covered by canvas to protect it from rain, find the area of canvas required.* **[2010]**

Sol. Here, $r = \frac{8}{2}$ m = 4 m, $h = 3$ m

$\therefore l = \sqrt{h^2 + r^2} = \sqrt{9 + 16}$ m = 5 m

Volume of the heap = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 \text{ m}^3$

$= \frac{352}{7} \text{ m}^3 = 50\frac{2}{7} \text{ m}^3$

Area of canvas required to cover the heap

= curved surface area of the heap

$= \pi rl = \frac{22}{7} \times 4 \times 5 \text{ m}^2 = \frac{440}{7} \text{ m}^2 = 62\frac{6}{7} \text{ m}^2$

PRACTICE EXERCISE 13.7A

Choose the correct option (Q 1 – 6) :

1 Mark Questions

1. The volume of the largest cone which can be cut from a cube of edge 9 cm is :

(*a*) $\frac{27\pi}{4}$ cm^3 (*b*) $\frac{243\pi}{4}$ cm^3

(*c*) 729π cm^3 (*d*) $\frac{243\pi}{4}$ cm^3

2. Volume of the cone of base radius r and slant height $r\sqrt{2}$ is : **[Imp.]**

(*a*) $\frac{1}{3}\pi r^3$ (*b*) $\frac{\sqrt{2}}{3}\pi r^3$

(*c*) $\sqrt{2}\pi r^3$ (*d*) $\frac{\pi r^3}{2\sqrt{3}}$

3. The circumference of the base of a cone of height 6 cm is 88 cm. The volume of the cone is :

(*a*) 1200 cm^3 (*b*) 1225 cm^3

(*c*) 1232 cm^3 (*d*) 1300 cm^3

4. A cylinder and a cone have the same base and same height. The ratio of their volumes (in the same order) is : **[2010]**

(*a*) 3 : 1 (*b*) 4 : 3

(*c*) 1 : 3 (*d*) 3 : 4

5. A cylinder and a cone have the same base radius. If their volumes are equal, then ratio of their heights is :

(*a*) 2 : 1 (*b*) 1 : 2

(*c*) 1 : 3 (*d*) 1 : 4

6. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is :

(*a*) 4.2 cm (*b*) 2.1cm

(*c*) 2.4 cm (*d*) 1.6 cm

2 Marks Questions

7. Find the volume of a conical tin having radius of the base as 30 cm and its slant height as 50 cm.

8. Find the volume of the largest right circular cone that can be fitted in a cube whose edge is $2r$.

3 Marks Questions

9. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface area of the solid thus generated. **[V. Imp.]**

10. There are two cones. The curved surface area of one cone is twice that of the other. The slant height of the later is twice that of the former. Find the ratio of their radii.

11. The area of base of a cone is 78.5 cm^2. If the height of the cone is 12 cm, find its volume and the curved surface area.

4 Marks Questions

12. A cone of height 24 cm has a curved surface area 550 cm^2. Find its volume.

13. An open cylindrical vessel of internal diameter 49 cm and height 64 cm stands on a platform. Inside it a solid metallic right circular cone of radius 5.25 cm and height 12 cm is placed. Find the volume of water to fill the remaining space of the vessel. **[HOTS]**

13.8 VOLUME OF A SPHERE

1. Volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

2. Volume of a hemisphere of radius r is $\frac{2}{3}\pi r^3$.

TEXTBOOK'S EXERCISE 13.8

Q.1. *Find the volume of a sphere whose radius is*

(i) 7 *cm* *(ii)* 0.63 *m*

Sol. (i) Here, $r = 7$ cm

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3 = 1437\frac{1}{3} \text{ cm}^3$

(ii) Here, $r = 0.63$ m

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.63)^3 \text{ m}^3 = 1.05 \text{ m}^3$ (approx)

Q.2. *Find the amount of water displaced by a solid spherical ball of diameter*

(i) 28 *cm* **[2011 (T-II)]** *(ii)* 0.21 *m*

Sol. (i) Here, $r = \frac{28}{2}$ cm = 14 cm

Volume of water displaced by the spherical ball $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 \text{ cm}^3$

$= 11498\frac{2}{3} \text{ cm}^3$

(ii) Here, $r = \frac{0.21}{2}$ m = 0.105 m

Volume of the water displaced by the spherical ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0.105 \times 0.105 \times 0.105 \text{ m}^3$
$= 0.004851 \text{ m}^3$

Q.3. *The diameter of a metallic ball is* 4.2 *cm. What is the mass of the ball, if the density of the metal is* 8.9 *g per cm*^{3}? **[2010]**

Sol. Here, $r = \frac{4.2}{2}$ cm = 2.1 cm

Volume of the ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 \text{ cm}^3 = 38.808 \text{ cm}^3$

Density of the metal = 8.9 g/cm^3

$\therefore$ Mass of the ball = 8.9 × 38.808 g
= 345.39 g (approx)

Q.4. *The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?* **[2010]**

Sol. Let diameter of the earth be $2r$. Then radius of the earth = r

So, diameter of the moon $= \frac{2r}{4} = \frac{r}{2}$

$\Rightarrow$ Radius of the moon $= \frac{r}{4}$

Volume of the earth $= \frac{4}{3}\pi r^3$ ... (i)

Volume of the moon $= \frac{4}{3}\pi\left(\frac{r}{4}\right)^3$ ... (ii)

$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\left(\frac{r}{4}\right)^3}$ [From (i) and (ii)]

$\Rightarrow \frac{r^3}{\frac{r^3}{64}} = \frac{64}{1} = 64$

$\Rightarrow$ Volume of the moon

$= \frac{1}{64} \times$ volume of the earth

Hence, volume of the moon is $\frac{1}{64}$ of volume of the earth.

Q.5. *How many litres of milk can a hemispherical bowl of diameter* 10.5 *cm hold?*

Sol. Here, $r = \frac{10.5}{2}$ cm = 5.25 cm

Volume of the hemispherical bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25 \text{ cm}^3$

$= 303 \text{ cm}^3$ (approx)

Hence, the hemispherical bowl can hold $\frac{303}{1000}$ litres = 0.303 liters of milk.

Q.6. *A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.* **[2011 (T-II)]**

Sol. Here, inner radius of the tank (r) = 1 m

Thickness of the iron sheet = 1 cm = 0.01 m

$\therefore$ External radius of the tank (R)

= (1 + 0.01) m = 1.01 m

Volume of the iron used to make the tank

$$= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7} \times [(1.01)^3 - 1^3]\ m^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.030301\ m^3 = 0.06348\ m^3$$

Q.7. *Find the volume of a sphere whose surface area is 154 cm*^{2}. **[2011 (T-II)]**

Sol. Here, $4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4} \Rightarrow r = \frac{7}{2} = 3.5\ cm$$

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5\ cm^3 = 179\frac{2}{3}\ cm^3$$

Q.8. *A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs 498.96. If the cost of whitewashing is Rs 2.00 per square metre, find the* **[2011 (T-II)]**

(i) inside surface area of the dome,

(ii) volume of the air inside the dome.

Sol. (i) Inner surface of the dome

$$= \frac{\text{Total cost}}{\text{Cost of whitewashing per m}^2}$$

$$= \frac{498.96}{2}\ m^2 = 249.48\ m^2$$

(ii) Let radius of the dome be r m.

Then, $2\pi r^2 = 249.48$

$$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48$$

$$\Rightarrow r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69 \Rightarrow r = 6.3\ cm$$

$\therefore$ Volume of the air inside the dome $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3\ m^3 = 523.9\ m^3$$

Q.9. *Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the*

(i) radius r′ of the new sphere.

(ii) ratio of S and S′.

Sol. (i) Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$

$\therefore$ Volume of 27 such spheres $= 27 \times \frac{4}{3}\pi r^3$

$= 36\pi r^3$

Volume of the sphere with radius $r' = \frac{4}{3}\pi r'^3$

$$\therefore 36\pi r^3 = \frac{4}{3}\pi r'^3 \Rightarrow 27r^3 = r'^3$$

$$\Rightarrow r' = \sqrt[3]{27r^3} \Rightarrow r' = 3r$$

(ii) Surface area (S) of the sphere with radius r

$= 4\pi r^2$

Surface area (S′) of the sphere with radius r'

$= 4\pi r'^2 = 4\pi(3r)^2 = 36\pi r^2$

$$\therefore \quad \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1 : 9$$

Q.10. *A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm*^{3}*) is needed to fill this capsule?*

Sol. Here, $r = \frac{3.5}{2}$ mm = 1.75 mm

Volume of the capsule $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 1.75\ mm^3$$

= 22.46 mm^3 (approx)

Hence, 22.46 mm^3 of medicine is needed to fill the capsule.

OTHER IMPORTANT QUESTIONS

Q.1. *The radius of a sphere and the edge of a cube are equal. The ratio of their volumes is :* **[2011 (T-II)]**

(*a*) $4\pi : 3$ (*b*) $4 : 3$
(*c*) $3 : 4$ (*d*) $4\pi : 1$

Sol. (*a*) Required ratio = $\dfrac{\frac{4}{3}\pi r^3}{r^3} = \dfrac{4\pi}{3} = 4\pi : 3$

Q.2. *The volume of a sphere of diameter r is:* **[2010]**

(*a*) $\frac{4}{3}\pi r^3$ (*b*) $\frac{8}{3}\pi r^3$
(*c*) $\frac{1}{6}\pi r^3$ (*d*) $\frac{1}{3}\pi r^3$

Sol. (*c*) Volume of the sphere

$= \frac{4}{3} \times \pi \times \left(\frac{r}{2}\right)^3 = \frac{\pi r^3}{6}$

Q.3. *If the volume and surface area of a sphere are numerically equal, then, its radius is:* **[2010]**

(*a*) 2 units (*b*) 3 units
(*c*) 4 units (*d*) 5 units

Sol. (*b*) We have, $\frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3$

Q.4. *A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.* **[V. Imp.]**

Sol. Required ratio

$= \frac{1}{3}\pi r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r$

$= \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$

Q.5. *The largest sphere is carved out of a cube of side 7 cm. Find the volume of the sphere.*

Sol. Radius of the sphere = $\frac{7}{2}$ cm

$\therefore$ Volume of the sphere

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$ cm^3

$= 179.66$ cm^3.

Q.6. *The diameter of a metallic ball is* 21 cm. *What is the mass of the ball, if the density of the metal is* 5 *gm per cm*^{3}*?* **[2011 (T-II)]**

Sol. Radius of the ball = $\frac{21}{2}$ cm = 10.5 cm

Volume of the ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5 = 4851$ cm^3

$\therefore$ Mass of the ball = 5×4851 g = 24255 g

Q.7. *The surface area of a sphere of radius* 5 *cm is five times the area of the curved surface of a cone of radius* 4 *cm. Find the height and the volume of the cone.* **[HOTS]**

Sol. Surface area of the sphere

$= 4\pi \times 5 \times 5$ cm^2

Curved surface area of the cone

$= \pi \times 4 \times l$ cm^2

where l is the slant height of the cone.

According to the statement $4\pi \times 5 \times 5$

$= 5 \times \pi \times 4 \times l \Rightarrow l = 5$ cm.

Now, $l^2 = h^2 + r^2$. Therefore,

$(5)^2 = h^2 + (4)^2$

where h is the height of the cone.

$\Rightarrow (5)^2 - (4)^2 = h^2$

$\Rightarrow (5+4)(5-4) = h^2 \Rightarrow 9 = h^2 \Rightarrow h = 3$ cm

Volume of cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3$ cm^3

$= \frac{22 \times 16}{7}$ cm^3 = $\frac{352}{7}$ cm^3 = 50.29 cm^3

Q.8. *A solid spherical ball of diameter* 4.2 *cm is completely immersed in water. How much water is displaced?* **[2011 (T-II)]**

Sol. Volume of water displaced = Volume of the sphere = $\frac{4}{3} \times \frac{22}{7} \times (2.1)^3$ cm^3 = 38.8 cm^3.

Q.9. *Find the volume, curved surface area and total surface area of a solid hemisphere of diameter 7 cm.* **[2010]**

Sol. $r = \frac{7}{2}$cm

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$ cm^3

$= \frac{539}{6}$ cm^3 = $89\frac{5}{6}$ cm^3

Curved surface area of the hemisphere

$= 2\pi r^2 = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ cm^2 = 77 cm^2

Total surface area of the hemisphere $= 3\pi r^2$

$= 3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ cm^2 = 115.5 cm^2

Q.10. *The solid sphere of radius 4 cm is melted and then cast into smaller spherical balls of diameter 0.8 cm. Find the number of these balls.* **[2010]**

Sol. Volume of the larger sphere $= \frac{4}{3}\pi \times 4^3$ cm^3

Volume of 1 smaller ball $= \frac{4}{3}\pi \times (0.4)^3$ cm^3

Let n number of balls are formed.

Then, $n \times \frac{4}{3}\pi \times (0.4)^3 = \frac{4}{3}\pi \times 4^3$

$\Rightarrow n = 1000$

Q.11. *Find the surface area of a sphere whose volume is $\frac{99}{7}$ cm^3.* **[2010]**

Sol. We have, $\frac{4}{3}\pi r^3 = \frac{99}{7}$

$\Rightarrow r^3 = \frac{99}{7} \times \frac{3 \times 7}{4 \times 22} = \frac{27}{8} \Rightarrow r = \frac{3}{2} = 1.5$ cm

Surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 1.5 \times 1.5$ cm^2 $= \frac{198}{7}$ cm^2

Q.12. *A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.* **[Imp.]**

Sol. Radius of the sphere = 2 cm

Volume of the sphere $= \frac{4}{3}\pi \times 2^3$ cm^3

$= \frac{32\pi}{3}$ cm^3

Volume of the cube = 4^3 cm^3 = 64 cm^3

$\therefore$ Required volume $= \left(64 - \frac{32\pi}{3}\right)$ cm^3

= 30.47 cm^3

Q.13. *A hemispherical bowl of internal radius 9 cm is full of a liquid. This liquid is to be filled into small cylindrical bottles of diameter 3 cm and height 4 cm each. Find the minimum number of bottles required to empty the bowl.* **[2010]**

Sol. Radius of the bowl = 9 cm

Volume of the bowl $= \frac{2}{3}\pi \times 9^3$ cm^3

Radius of 1 bottle = 1.5 cm

Height of 1 bottle = 4 cm

Volume of 1 bottle $= \pi \times (1.5)^2 \times 4$ cm^3

Let n bottles are needed.

Then, $n \times \pi \times (1.5)^2 \times 4 = \frac{2}{3}\pi \times 9^3$

$\Rightarrow n = \frac{2 \times 9 \times 9 \times 9}{3 \times 1.5 \times 1.5 \times 4} = 54$

Hence, 54 cylindrical bottles are needed.

Q.14. *A sphere and a cube have the same surface area. Find the ratio of their volumes.* **[2010]**

Sol. Let the radius of the sphere be r and edge of the cube be a.

Then, surface area of the sphere $= 4\pi r^2$

And, the surface area of the cube $= 6a^2$

$\therefore 4\pi r^2 = 6a^2 \Rightarrow \left(\frac{r}{a}\right) = \sqrt{\frac{3}{2\pi}}$ ... (i)

Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of the cube $= a^3$

$\therefore$ Required ratio = $\dfrac{\frac{4\pi r^3}{3}}{a^3} = \dfrac{4\pi}{3}\left(\dfrac{r}{a}\right)^3$

$= \dfrac{4\pi}{3} \cdot \dfrac{3}{2\pi} \times \sqrt{\dfrac{3}{2\pi}} = \dfrac{2\sqrt{3}}{\sqrt{2\pi}} = \sqrt{6} : \sqrt{\pi}$

Q.15. *The surface area of a sphere is* 2464 *cm*^{2}. *Find its volume.* **[2010]**

Sol. We have, $4\pi r^2 = 2464 \Rightarrow r^2 = \dfrac{2464 \times 7}{4 \times 22}$

$= 196 \Rightarrow r = 14$ cm

$\therefore$ Volume of the sphere $= \dfrac{4}{3}\pi \times (14)^3$ cm^3

$= \dfrac{4}{3} \times \dfrac{22}{7} \times 14 \times 14 \times 14 \text{ cm}^3 = \dfrac{34496}{3}$ cm^3

$= 11498\dfrac{2}{3}$ cm^3

Q.16. *The volume of two spheres are in the ratio* 64 : 27. *Find their radii, if the sum of their radii is* 21 *cm.* **[2010]**

Sol. Let the radius of the first sphere be r cm. Then,

Radius of another sphere = $(21 - r)$ cm

Volume of first sphere $= \dfrac{4}{3}\pi \times r^3$

Volume of another sphere $= \dfrac{4}{3}\pi \times (21 - r)^3$

Ratio of their volumes $= \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (21-r)^3}$

$\Rightarrow \dfrac{64}{27} = \dfrac{r^3}{(21-r)^3}$

$\Rightarrow \dfrac{4}{3} = \dfrac{r}{21-r} \Rightarrow 3r = 84 - 4r$

$\Rightarrow 7r = 84 \Rightarrow r = \dfrac{84}{7} = 12$

$\therefore$ Hence, radii of the spheres are 12 cm and (21 – 12) cm = 9 cm

Q.17. *Three solid spheres of iron whose diameters are* 2 *cm,* 12 *cm and* 16 *cm respectively are melted into a spheres. Find the radius of the new sphere.* **[2010, 2011 (T-II)]**

Sol. Radii of the three spheres are 1 cm, 6 cm and 8 cm.

Volume of these spheres are $\dfrac{4}{3}\pi \times 1^3$ cm^3, $\dfrac{4}{3}\pi \times 6^3$ cm^3 and $\dfrac{4}{3}\pi \times 8^3$ cm^3 respectively.

Let the radius of the new sphere be R. Then,

$\dfrac{4}{3}\pi \times 1^3 + \dfrac{4}{3}\pi \times 6^3 + \dfrac{4}{3}\pi \times 8^3 = \dfrac{4}{3}\pi \times R^3$

$\Rightarrow 1 + 216 + 512 = R^3$

$\Rightarrow 729 = R^3 \Rightarrow R = 9$

Hence, radius of the new sphere is 9 cm.

Q.18. *The total surface area of a solid hemisphere is* 1848 *cm*^{2}. *Find the volume of the hemisphere.* **[2010]**

Sol. We have $3\pi r^2 = 1848$

$\Rightarrow r^2 = \dfrac{1848 \times 7}{3 \times 22} = 196 \Rightarrow r = 14$ cm

Volume of the hemisphere

$= \dfrac{2}{3}\pi \times (14)^3$ cm^3

$= \dfrac{2}{3} \times \dfrac{22}{7} \times 14 \times 14 \times 14$ cm^3

$= \dfrac{17248}{3}$ cm$^3 = 5749\dfrac{1}{3}$ cm^3

PRACTICE EXERCISE 13.8A

Choose the correct option (Q 1 – 5) :

1 Mark Questions

1. 27 solid iron spheres each of radius r, are melted to form a new sphere. The radius of the new sphere is :

(*a*) $2r$ (*b*) $3r$

(*c*) $4r$ (*d*) $7r$

2. The volumes of two spheres are in the ratio 64 : 27. The ratio of their radii is equal to : **[Imp.]**

(*a*) 4 : 3 (*b*) 3 : 4
(*c*) 16 : 9 (*d*) 16 : 27

3. The radius of a sphere is $2r$, then its volume will be :

(*a*) $\frac{4}{3}\pi r^3$ (*b*) $4\pi r^3$

(*c*) $\frac{8\pi r^3}{3}$ (*d*) $\frac{32}{3}\pi r^3$

4. Volume of a sphere is $288\,\pi$ cm^3. Its radius is :

(*a*) 5 cm (*b*) 6 cm
(*c*) 7 cm (*d*) 10.5 cm

5. Radius of a sphere has been increased from r to $3r$. The volume of the new sphere becomes :

(*a*) 3 times (*b*) 6 times
(*c*) 9 times (*d*) 10 times

2 Marks Questions

6. The volume of a sphere is equal to two-thirds of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere. Is this statement true ? Justify your answer.

7. How many lead shots each of radius 1 cm can be made from a sphere of radius 8 cm ?

8. If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere ?

9. A sphere is inscribed in a cube. Find the ratio of the volume of the cube to the volume of the sphere. **[V. Imp.]**

10. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

3 Marks Questions

11. A solid sphere of radius 3 cm is melted and then costed into small spherical balls each of diameter 0.6 cm. Find the number of balls thus, obtained. (use $\pi = \frac{22}{7}$) **[2011 (T-II)]**

12. If the radius of a sphere is increased by 10%, by how much per cent will its volume increase ?

13. An iron sphere of radius 5 cm is melted and smaller spheres each of radius 2.5 cm are made. How many smaller spheres are obtained ?

14. A hemispherical vessel full of water is emptied in a cone. The radii of the vessel and the cone are 12 cm and 8 cm respectively. Find the height of the water in the cone.

15. What is the least number of solid metallic spheres of 6 cm in diameter that should be melted and recast to form a solid metal cylinder whose height is 45 cm and diameter 4 cm? **[Imp.]**

16. Two solid spheres made of the same metal have weights 5920 g and 740 g respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

4 Marks Questions

17. The volumes of two spheres are in the ratio 64 : 27. Find the ratio of their surface areas. **[2011 (T-II)]**

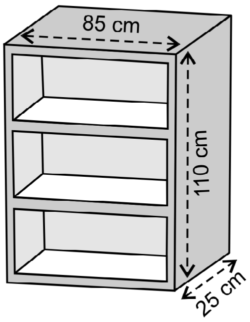
18. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height ?

19. A sphere, a cylinder and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere. Find the ratio of their volumes. **[HOTS]**

20. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Find the volume of the water pumped into the tank.

TEXTBOOK'S EXERCISE 13.9 (OPTIONAL)

Q.1. *A wooden book-shelf has external dimensions as follows : Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per* cm^2 *and the rate of painting is 10 paise per* cm^2*, find the total expenses required for polishing and painting the surface of the bookshelf.*



Sol. Here, external dimensions of the bookshelf are :

L = 110 cm, B = 85 cm, H = 25 cm

Thickness of the plank = 5 cm

Internal dimensions of the bookshelf are :

$l = (110 - 5 - 5)$ cm = 100 cm,

$b = (85 - 5 - 5)$ cm = 75 cm,

$h = (25 - 5)$ cm = 20 cm

External surface area of the bookshelf

= LB + 2 (BH + HL)

$= 110 \times 85\ cm^2 + 2(85 \times 25 + 25 \times 110)\ cm^2$

$= (9350 + 9750)\ cm^2 = 19100\ cm^2$

Surface area of the border

$= (4 \times 75 \times 5 + 110 \times 5 \times 2)\ cm^2$

$= (1500 \times 1100)\ cm^2 = 2600\ cm^2$

$\therefore$ Total surface area to be polished

$= (19100 + 2600)\ cm^2 = 21700\ cm^2$

$\therefore$ Cost of polishing the outer surface

$= \text{Rs}\ \dfrac{21700 \times 20}{100} = \text{Rs}\ 4340$ ... (i)

Inner surface area of the bookshelf

$= lb + 2(bh + hl)$

$= 100 \times 75\ cm^2 + 2\ (75 \times 20 + 20 \times 100)\ cm^2$

$= 7500\ cm^2 + 2\ (1500 + 2000)\ cm^2$

$= (7500 + 7000)\ cm^2 = 14500\ cm^2$

Surface area of the two racks

$= 4 \times 75 \times 20\ cm^2 = 6000\ cm^2$

Inner surface area covered by the racks

$= (75 \times 5 \times 2 + 20 \times 5 \times 4)\ cm^2$

$= (750 + 400)\ cm^2 = 1150\ cm^2$

$\therefore$ Total surface area to be painted

$= (14500 + 6000 - 1150)\ cm^2$

$= 19350\ cm^2$

$\therefore$ Cost of painting the inner surface

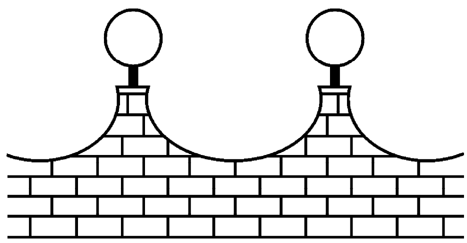
$= \text{Rs}\ \dfrac{19350 \times 10}{100} = \text{Rs}\ 1935$ ... (ii)

From (i), and (ii), we have,

Total expenses required for polishing and painting the surface of the bookshelf

= Rs (4340 + 1935) = Rs 6275

Q.2. *The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per* cm^2 *and black paint costs 5 paise per* cm^2.



Sol. Radius of a sphere $= \dfrac{21}{2}$ cm = 10.5 cm

Surface area of a sphere $= 4\pi r^2$

$= 4 \times \dfrac{22}{7} \times 10.5 \times 10.5\ cm^2 = 1386\ cm^2$

Area of the base of the cylinder (support)

$= \pi R^2 = \pi \times (1.5)^2$

$= \frac{22}{7} \times 1.5 \times 1.5 \text{ cm}^2 = 7.07 \text{ cm}^2$

Area of a sphere to be painted silver

$= (1386 - 7.07) \text{ cm}^2 = 1378.93 \text{ cm}^2$

Area of spheres to be painted silver

$= 8 \times 1378.93 \text{ cm}^2$

$\therefore$ Cost of painting the spheres

$= \text{Rs } \frac{8 \times 1378.93 \times 25}{100}$

= Rs 2757.86

Curved surface area of a cylinder (support)

$= 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$

Curved surface area of 8 supports

$= 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$

Cost of painting the supports

$= \text{Rs } 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \times \frac{5}{100}$

= Rs 26.40

Total cost of paint = Rs (2757.86 + 26.40)

= Rs 2784.26

Q.3. *The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?*

Sol. Let originally the diameter of the sphere be $2r$.

Then, radius of the sphere = r

Surface area of the sphere = $4\pi r^2$ ... (i)

New diameter of the sphere

$= 2r - 2r \times \frac{25}{100} = \frac{3r}{2}$

$\therefore$ New radius of the sphere = $\frac{3r}{4}$

Surface area of the new sphere

$= 4\pi \left(\frac{3r}{4}\right)^2 = \frac{9\pi r^2}{4}$

Decrease in surface area

$= 4\pi r^2 - \frac{9\pi r^2}{4} = \frac{7\pi r^2}{4}$

Per cent decrease $= \dfrac{\frac{7\pi r^2}{4} \times 100}{4\pi r^2}$

$= \frac{7}{16} \times 100 = \frac{175}{4} = 43.75$

Hence, the surface area decreases by 43.75%

B. FORMATIVE ASSESSMENT

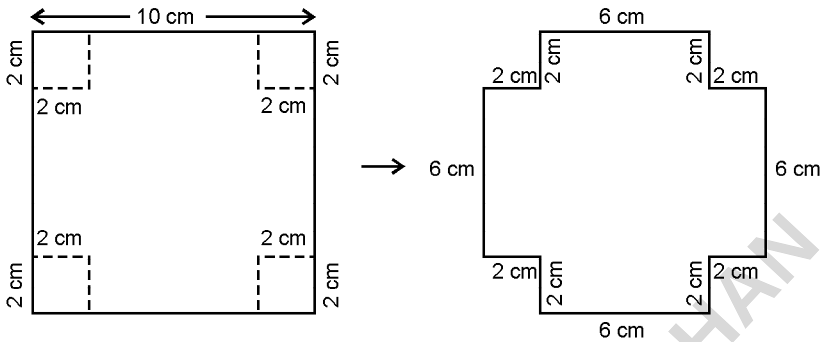
Project – 1

Objective : To make an open box using a square sheet of paper by cutting four equal squares from its corners. Also, to investigate what should be the size of the squares cut to have the maximum volume of the box.

Materials Required : Some thick square sheets of paper of length 10 cm, sellotape, a pair of scissors, geometry box, etc.

Procedure : Take a thick square sheet of paper of side 10 cm. Cut out squares from the corners in all possible square sizes, in half-cm units from 0.5 cm to 3 cm. Fold the remaining shapes using sellotape in each case to make open boxes. Calculate the length, breadth, height and consequently the volume of each box so formed. Observe which box has maximum volume. Graphically plot the data using size of the squares cut against the volume of the box formed.

Draw all the necessary diagrams. One diagram is shown.



Observations :

Prepare a table to record the length, breadth and height of each box.

Box	Length (l)	Breadth (b)	Height (h)
(i)	9 cm	9 cm	0.5 cm
(ii)	8 cm	8 cm	1 cm
(iii)	7 cm	7 cm	1.5 cm
(iv)	6 cm	6 cm	2 cm
(v)	5 cm	5 cm	2.5 cm
(vi)	4 cm	4 cm	3 cm

The length of each side of the given square = 10 cm.

If the length of the side of the square cut be x cm, then, the length of the box = the breadth of the box = $(10 - 2x)$ cm

And, the height of the box = x cm.

Calculate the volume of each box from the data given above.

Draw a graph by taking length of the square cut (height of the box) along x-axis and volume of the box along y-axis.

Conclusion : Write the conclusion you draw from the project.

Project – 2

Objective : To form cube, cuboids and cones using flat cut outs and to obtain formulae for their total surface areas.

Materials Required : Some thick sheets of graph paper with dimensions 30 cm × 20 cm, sellotape, geometry box, a pair of scissors, etc.

Project Overview : The world around us is made up of geometric shapes. Some of the objects have shapes like cube, cuboid, cylinder, cone, sphere and pyramid. These are called solids or 3-dimensional shapes. It is easy to make a solid shape from a piece of card by first drawing a net of the faces of the solid. The net is made up of different plane shapes (rectangles, squares, triangles, circles). When the net is cut out and the faces of the net folded along the edges, then a solid is formed.

Procedure :

1. On a graph paper, draw a net of a cube of edge 4 cm as shown in the figure.

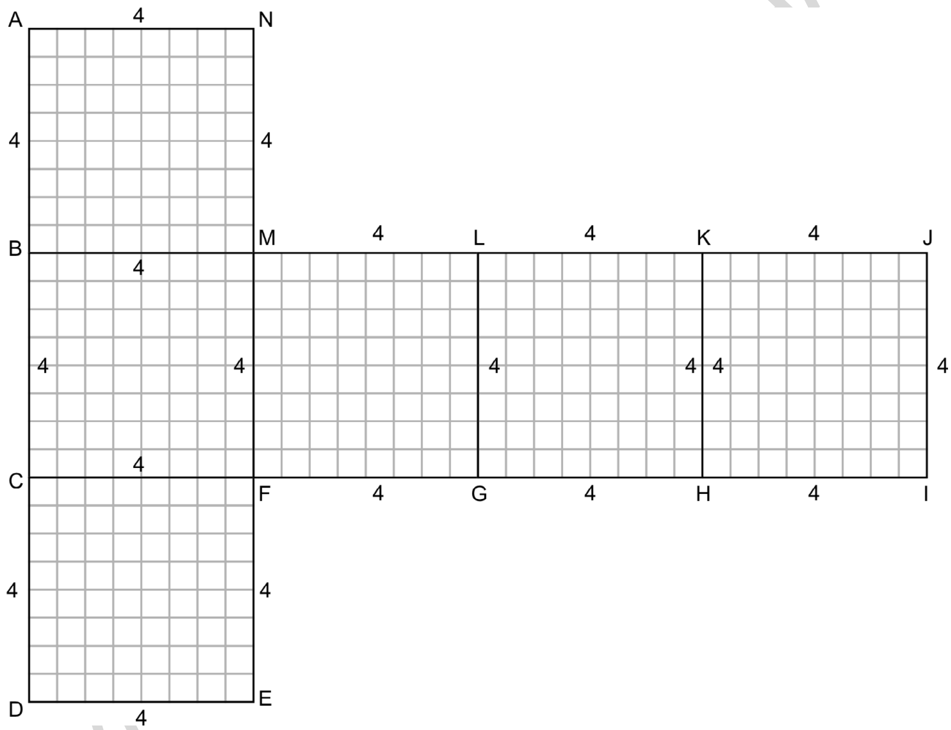


Figure-1

2. Cut out the net and fold it along the edges to get a box (cube) of edge 4 cm as shown in the figure.

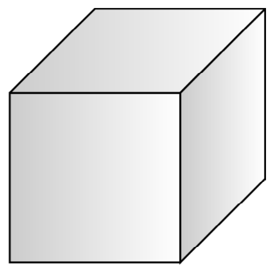


Figure-2

3. On a graph paper, draw a net of a cuboid of dimensions 5 cm × 3 cm × 2 cm as shown in the figure.

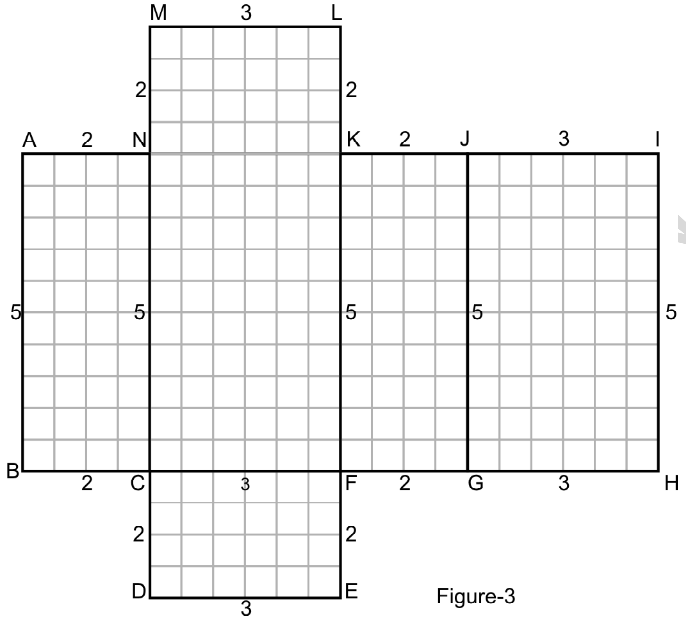


Figure-3

4. Cut out the net and fold it along the edges to get a box (cuboid) as shown in the figure.

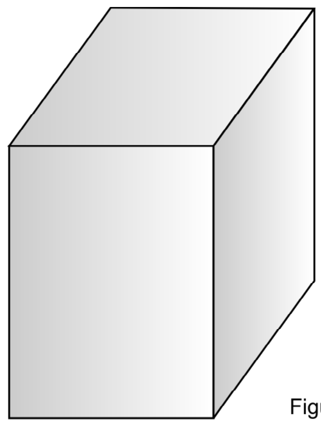


Figure-4

5. Draw a circle of radius 7 cm and from it cut out a quadrant of the circle as shown in the figure.

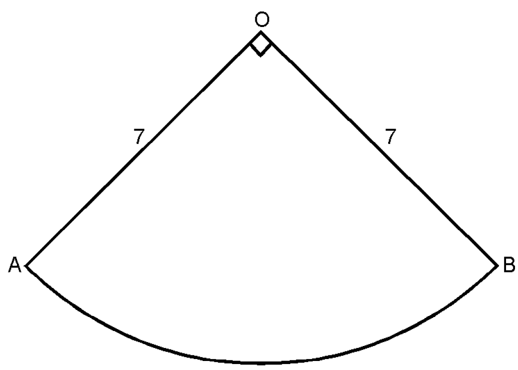


Figure-5

6. Fold the quadrant (sector) of the circle so that the ends of the arc meet. It gives a hollow cone without base as shown in the figure.

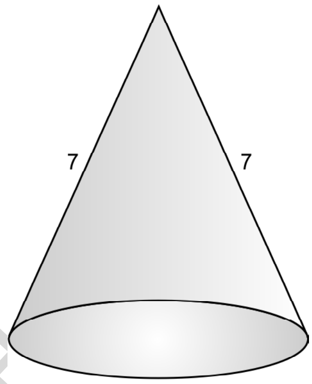


Figure-6

7. Place the cone obtained above over a white sheet of paper and draw the outline of its base. The outline is a circle. Cut out the circle and using sellotape, paste it at the base of the hollow cone to get a closed conical shape.

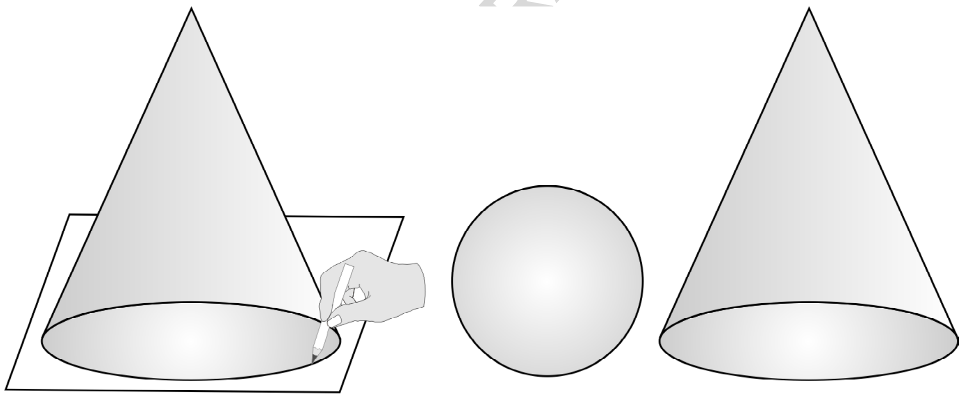


Figure-7

Observations :

1. The cubical box has six square faces. The length of the side of each square is 4 cm.
2. The cuboid has six rectangular faces.
   Two faces are of dimensions 5 cm × 3 cm
   Two faces are of dimensions 3 cm × 2 cm
   And two faces are of dimensions 5 cm × 2 cm.
3. The length of the arc of the quadrant (sector) of the circle is equal to the circumference of the base (circle) of the cone.
4. The area of the quadrant (sector) of the circle is equal to the curved surface area of the cone.

Computation :

For Cube [Figs. 1 and 2]

The area of ABMN = 4^2 cm^2

$\therefore$ The area of all the faces = (6×4^2) cm^2

Thus, the total surface area of the cube = (6×4^2) cm^2

If a be the length of one edge of any cube, then the area of its one face = a^2

$\therefore$ The area of 6 faces = $6a^2$

$\therefore$ The total surface area of a cube of edge $a = 6a^2$

For Cuboid [Figs. 3 and 4]

The area of CFKN = 5×3 cm^2.

$\therefore$ The area of CFKN and GHIJ = $2 \times (5 \times 3)$ cm^2

Similarly, the area of LMNK and CDEF = $2 \times (3 \times 2)$ cm^2

And, the area of ABCN and FGJK = $2 \times (2 \times 5)$ cm^2

Thus, the total surface area of the cuboid = area of CFKN and GHIJ + area of LMNK and CDEF + area of ABCN and FGJK = $[2 \times (5 \times 3) + 2 \times (3 \times 2) + 2 \times (2 \times 5)]$ cm^2

$= 2[(5 \times 3) + (3 \times 2) + (2 \times 5)]$ cm^2

Thus, if l, b and h be the length, breadth and height of any cuboid, then

The total surface area of the cuboid = $2(lb + bh + hl)$.

For Cone [Figs 5, 6 and 7]

The length of arc AB = Circumference of the base of the cone

$\Rightarrow \dfrac{2\pi \times 7}{4} = 2\pi r$, where r is the base radius of the cone.

$\Rightarrow r = \dfrac{7}{4}$ cm = 1.75 cm

The area of the sector (quadrant) $= \dfrac{\pi \times 7^2}{4}\text{ cm}^2 = \pi \times \dfrac{7}{4} \times 7\text{ cm}^2$

$= \pi \times 1.75 \times 7$ cm^2

$\Rightarrow$ The curved surface area of the cone $= \pi \times 1.75 \times 7$ cm^2

$= \pi \times$ radius of the base $\times$ slant height

Also, the area of the base of the cone = area of the circle with radius 1.75 cm

$= \pi \times (1.75)^2$ cm^2

$= \pi \times$ (base radius)^2

Thus, total surface area of the cone = curved surface area + area of the base

$= \pi \times 1.75 \times 7$ cm$^2 + \pi \times (1.75)^2$ cm^2.

If r be the base radius and l be the slant of a cone, then the total surface area of the cone

$= \pi rl + \pi r^2$

Conclusion :

(i) The total surface area of a cube of edge $a = 6a^2$

(ii) The total surface area of a cuboid of length, breadth and height as l, b and h respectively

$= 2(lb + bh + hl)$

(iii) The total surface area of a cone of base radius r and slant height $l = (\pi rl + \pi r^2)$

$= \pi r(l + r)$

ANSWERS

Practice Exercise 13.1A

1. (*a*) **2.** (*d*) **3.** (*a*) **4.** (*b*) **5.** (*a*) **6.** (*c*) **7.** $3\sqrt{11}$ cm **8.** 4 cm **9.** 504 cm^2 **10.** 7 : 9 **11.** 792 cm^2 **12.** 7 : 10 **13.** 10 cm, 6 cm, 4 cm

Practice Exercise 13.2 A

1. (*d*) **2.** (*c*) **3.** (*c*) **4.** (*c*) **5.** (*b*) **6.** (*c*) **7.** False **8.** 880 cm^2 **9.** 10560 cm^2 **10.** 1 m **11.** 633.60 m^2 **12.** 739.2 *l*

Practice Exercise 13.3 A

1. (*a*) **2.** (*d*) **3.** (*c*) **4.** (*d*) **5.** (*b*) **6.** (*c*) **7.** 528 cm^2 **8.** 753.6 cm^2, 1205.76 cm^2 **9.** 165 cm^2 **10.** 137.5 m.

Practice Exercise 13.4 A

1. (*c*) **2.** (*a*) **3.** (*c*) **4.** (*a*) **5.** (*d*) **6.** 3.5 cm **7.** 154 cm^2 **8.** 173.25 cm^2, 259.875 cm^2 **9.** 2 : 3 **10.** 300%

Practice Exercise 13.5 A

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*d*) **5.** (*b*) **6.** 3375 cm^3 **7.** 1 : 8 **8.** 5 m **9.** 30 cm, 20 cm, 15 cm **10.** Rs 640

Practice Exercise 13.6 A

1. (*c*) **2.** (*c*) **3.** (*c*) **4.** (*c*) **5.** (*c*) **6.** (*b*) **7.** False **8.** 3080 cm^3 **9.** 38500 cm^3 **10.** 30 hours, 41 min, 20 seconds **11.** 1600 **12.** 191 cm **13.** 46.2 m **14.** 2.5 cm, 2 cm

Practice Exercise 13.7 A

1. (*d*) **2.** (*a*) **3.** (*c*) **4.** (*a*) **5.** (*c*) **6.** (*b*) **7.** 37680 cm^3 **8.** $\frac{2}{3}\pi r^3$ **9.** 301.7 cm^3, 188.5 cm^2 **10.** 4 : 1 **11.** 314 cm^3, 204.1 cm^2 **12.** 1232 cm^3 **13.** 120389.5 cm^3

Practice Exercise 13.8 A

1. (*b*) **2.** (*a*) **3.** (*d*) **4.** (*b*) **5.** (*c*) **6.** True **7.** 512 **8.** 1 : 8 **9.** 6 : π **10.** 36 m **11.** 1000 balls **12.** 33.3 **13.** 8 **14.** 54 cm **15.** 5 **16.** 5 cm **17.** 16 : 9 **18.** 50% **19.** 2 : 3 : 1 **20.** 668.6 m^3